Homework 5 for "Convex Optimization"

Zaiwen Wen
Beijing International Center for Mathematical Research
Peking University

November 21, 2021

1 Algorithms for the group LASSO problem

Consider the group LASSO problem

$$\min_{x \in \mathbb{R}^{n \times l}} \quad \frac{1}{2}\|Ax - b\|_F^2 + \mu\|x\|_{1,2} \tag{1.1}$$

where the data $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^{m \times l}$ and $\mu > 0$ are given, and

$$\|x\|_{1,2} = \sum_{i=1}^{n} \|x(i, 1:l)\|_2.$$

Note that $x(i, 1:l)$ is the i-th row of the matrix x. Here, both x and b are matrices but they are written in small letters for the convenience of coding. The test data are generated as follows:

```
seed = 97006855;
ss = RandStream('mt19937ar','Seed',seed);
RandStream.setGlobalStream(ss);
n = 512;
m = 256;
A = randn(m,n);
k = round(n*0.1); l = 2;
A = randn(m,n);
p = randperm(n); p = p(1:k);
u = zeros(n,l);  u(p,:) = randn(k,l);
b = A*u;
mu = 1e-2;
```

See `http://bicmr.pku.edu.cn/~wenzw/courses/Test_group_lasso.m`

1. Solve (1.1) using CVX by calling different solvers mosek and gurobi.
2. First write down an equivalent model of (1.1) which can be solved by calling mosek and gurobi directly, then implement the codes.
3. First write down, then implement the following algorithms in Matlab (or Python):

(a) Subgradient method for the primal problem.
Read the subgradient method in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-sgm.pdf`

(b) Gradient method for the smoothed primal problem.
Read the smoothing technique in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/Smoothing.pdf`

(c) Fast (Nesterov/accelerated) gradient method for the smoothed primal problem.
Read the acceleration techniques in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/slides-fgrad.pdf`

(d) Proximal gradient method for the primal problem.
Read `http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-proxg.pdf`

(e) Fast proximal gradient method for the primal problem.
Read the acceleration techniques in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/slides-fgrad.pdf`

(f) Augmented Lagrangian method for the dual problem.
Read the augmented Lagrangian method in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-prox-point.pdf`

(g) Alternating direction method of multipliers for the dual problem.
Read the augmented Lagrangian method in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-prox-point.pdf`

(h) Alternating direction method of multipliers with linearization for the primal problem.
Read the ADMM in `http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-admm.pdf`.
Read the ADMM with a single gradient (or proximal graident) step in pages 15 and 16 in
`http://bicmr.pku.edu.cn/~wenzw/opt2015/lect-admm-part2.pdf`

(i) Proximal point method for the dual problem

(j) Block coordinate method for the primal problem

4. Requirement:

(a) The interface of each method should be written in the following format

```
[x, iter, out] = method_name(x0, A, b, mu, opts);
```

Here, x0 is a given input initial solution, A, b and mu are given data, opts is a struct which stores the options of the algorithm, iter is the number of iterations when the termination condition of the algorithm is satisfied, out is a struct which saves all other output information.

(b) Compare the efficiency (cpu time) and accuracy (checking optimality condition) in the format as
`http://bicmr.pku.edu.cn/~wenzw/courses/Test_group_lasso.m`

(c) Prepare a report including

- detailed answers to each question
- numerical results and their interpretation

(d) Pack the report and all of your codes in one file named as "gl-StudentID-date.zip" and send it to TA: pkuopt@163.com

(e) If you get significant help from others on one routine, write down the source of references at the beginning of this routine.

(f) Due date: see `https://bicmr.pku.edu.cn/~wenzw/opt-2021-fall.html`