Homework 7 for "Algorithms for Big-Data Analysis"

Note: Please write up your solutions independently. If you get significant help from others, write down the source of references. A formal mathematical proof for all your claims is required.

1. Derive the dual optimization problem for

$$\begin{aligned} \min_{w,b,\xi} \quad & \frac{1}{2}\|w\|_2^2 + C_1 \sum_{i=1}^{n} \xi_i + C_2 \sum_{i=1}^{n} \xi_i^2 \\ \text{s.t.} \quad & y_i \cdot (x_i \cdot w + b) \geq 1 - \xi_i, \forall i = 1, \ldots, n \\ & \xi_i \geq 0, \forall i = 1, \ldots, n \end{aligned}$$

2. Properties of Submodular Functions

   (a) Prove that any non-negative submodular function is also subadditive, i.e. if $F : 2^X \to \mathbb{R}_+$ is submodular then $F(S \cup T) \leq F(S) + F(T)$ for any $S, T \subseteq X$. Here, $\mathbb{R}_+ = \{x \in \mathbb{R}, x \geq 0\}$.

   (b) Prove that a function $F : 2^X \to \mathbb{R}_+$ is submodular if and only if for any $S, T \subseteq X$, the marginal contribution function $F_S(T) = F(S \cup T) - F(S)$ is subadditive. (If the statement is not true, please either add a condition to make it correct or give a counterexample.)

3. Consider a graph (V, E), where V is the set of nodes and E is the set of edges. Let S be a subset of V and $V \backslash S$ be the complement of S. Define $f(S)$ be the number of edges $e = (u, v)$ such that $u \in S$ and $v \in V \backslash S$. Prove that $f(S)$ is submodular.

4. Exercise 3.4 in `http://incompleteideas.net/book/RLbook2020.pdf`

5. Exercise 3.23 in `http://incompleteideas.net/book/RLbook2020.pdf`