2017 BICMR 金融数学暑期学校
课程摘要

XVA analysis

Stéphane Crépey (Evry University)

Abstract (6 hours) : In the aftermath of the financial crisis, regulators launched a major effort of banking reform aimed at securing the financial system by raising collateralisation and capital requirements. Notwithstanding finance theories, according to which costs of capital and of funding for collateral are irrelevant to investment decisions, banks have introduced an array of XVA metrics to precisely quantify them. In particular, the cost of margins (MVA) and of capital (KVA) are emerging as metrics of key practical importance.
However, their economic foundation is still under debate. We present an XVA approach rooted in a capital structure model acknowledging the impossibility for a bank to replicate jump-to-default related cash flows. Because of this counterparty credit risk incompleteness, deals trigger wealth transfers from bank shareholders to bank creditors and shareholders need to set capital at risk.
On this basis we devise a theory of XVAs, whereby so-called contra-liabilities and cost of capital are sourced from bank clients at trade inceptions, on top of the fair valuation of counterparty credit risk, in order to compensate shareholders for wealth transfers and risk on capital. On the mathematical side, XVA analysis points out to BSDEs stopped before a random time solved in a framework of enlargement of filtration based on so called invariance times, which will also be introduced during the course.

Enlargement of filtration

Monique Jeanblanc (Evry University)

In this lecture, we shall explain, in a first part, enlargement of filtration results in a discrete time setting. This part has a pedagocical interest, since the results are easy to prove. In a second part, we shall give in continuous time, main results on initial and progressive enlargement. In a third part, we present application to finance: optimisation problems and arbitrages.

Continuous-state branching processes with immigration

Zenghu Li (Beijing Normal University)

Abstract (6 hours) : The classical Galton-Watson branching process is a stochastic process with discrete time and state arising from the statistical investigation of the extinction of names. The process can be constructed from positive integer-valued random variables using a recurrence formula, where the structure of trees is involved.

To describe the evolution of population with a large number of individuals, Feller (1951) introduced a branching process with continuous time and state. This model has been studied and generalized by many authors. A continuous time-state branching process with immigration (CBI-process) is a further generalization of the model taking into consideration the influence from the environments. The theory of CBI-processes has been developed rapidly in the past decades. The study has led to better understanding of many deep structures such as Brownian excursions, stochastic flows, planar maps and Levy trees. Those processes have also been used widely in mathematical finance as

models of interest rates, asset prices and so on.

In this mini course, we first introduce the CBI-process as a natural rescaling limit of the classical model with discrete time and state. We then present the contraction of the CBI-process in terms of stochastic equations given by Dawson and Li (2006, 2012) and Fu and Li (2010). In the third part of the course, we discuss some applications and open problems including distributional properties, estimation of parameters and related competition models.

Law of large Numbers under nonlinear expectation and phi-Max-mean algorithm of robust risk measures.

Shige Peng (Shandong University and Peking University)

Abstract (3 hours):
Kolmogorov's foundation of probability theory (1993), considered as the "Euclidean axiomatical system in probability theory", provides an important base to measure and pricing quantities of uncertainties in financial markets. But it is still a challenging problem to quantify uncertainties caused by the ambiguity of probability distributions. A crucial point here is that a probability measure corresponds only to an operation of linear expectation, and in fact it is the nonlinear expectation which provide us a powerful and quantitative mathematical tool to analyze and calculate such type of ambiguity.

In this course we firstly provide the foundation of nonlinear expectation theory, specifying some typical mathematical distribution models of random variables and stochastic processes.

We also present a new and very useful algorithm for computing robust risk measure with much weaker assumptions of the classical i.i.d. Our new algorithm is fundamentally based on a new law of large number under robust nonlinear expectations. This new result also reveal a deep relation among robust limit theory, robust statistics and quasilinear and fully nonlinear PDEs.

Real markets, microstructure, clusters with a Branching process point of view.

Simone Scotti (University of Paris 7)

Abstract (6 hours) : The analysis of real financial markets, along with the study of related economic time series, represents an increasing field of research and development of effective applications, from the stochastic processes point of views, as well as from the statistic and computational sides.

In this mini-course, we first recall the classical framework behind the modern and mathematically rigorous, theory of finance, starting from the treatment of continuous-path stochastic processes. Then, we focus on the microstructure of market data, discussing both limit and market orders, along with correlated liquidity problems.

In a second part, we will specialize our analysis on a particular feature of financial markets, namely the existence of jumps' cluster. It is worth to mention that the latter has been highlighted in recent literature in microstructure, showing to have a deep impact on real markets and representing a main research axis in the field. From a mathematical point-of-view, we will first introduce point processes and then Hawkes processes. We will show that the self-exciting structure of Hawkes processes can easily explain some features exhibited by financial data, e.g. the cluster effects.

In the third part, we will introduce the branching processes (CBI) showing that they can be seen as a

natural extension (marked versions) of Hawkes processes. We will show that this class of models has very nice properties from computational as well as from analytical point of view, particularly by exploiting Dawson-Li representation. Moreover, we will point out some unexpected features of the aforementioned approaches, such, e.g., the persistency of low interest rates.

Power Markets, Risk-Neutral Measures and Risk Premium in Lévy-based Models

Carlo Sgarra (Politecnico di Milano)

After illustrating the basic features of power markets, we shall present and discuss the most popular models proposed in the literature. Most of them are based on superposition of Independent Icrements Processes. We'll recall the fundamental properties of these processes and present the fundamental results related to derivatives pricing for power markets. In order to define a risk-neutral measure, we'll introduce the notion of the Esscher Transform, and we illustrate how to construct risk-neutral measures by using this tool. The first lecture will be spent in providing a review on the results available in the literature on power modelling and derivatives pricing. The following lecture will be devoted to a more specific investigation on the risk premium behavior in power markets and a systematic computation of the risk premium for the models proposed. The third lecture will present an alternative formulation of power price dynamics based on random fields and involving Continuous State Branching Processes (CBI). Some explicit formulas for the most common derivatives traded in the market will be provided, and we shall illustrate how a CBI-type dynamics can imply in a simple and natural way a very realistic risk premium term structure.

Arrow-Debreu Equilibria for Expected Utilities and Rank-Dependent Utilities

Jianming Xia (Chinese Academy of Science)

Abstract (4 hours): I will introduce the Arrow-Debreu equilibria first for expected utilities and then for rank-dependent utilities.

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Jingping Yang（Peking University）