Perspectives in Geometric Analysis-- Joint event of ANU-BICMR-PIMS-UW

Beijing International Center for Mathematical Research
Lecture Hall, Jia Yi Bing Building, 82# Jing Chun Yuan

Schedule and abstracts

Schedule:

6.27

Monday morning

8:30-9:30 **Xiaodong Wang**, Comparison and rigidity results on compact Riemannian manifolds with boundary

9:40-10:40 **Richard Bamler**, Ricci flows with bounded scalar curvature

11:00-12:00 **Jeff Streets**, Generalized Kahler-Ricci flow

Monday afternoon

2:00-3:00 **Rivière Tristan**, The Variations of Yang-Mills Lagrangian

3:10-4:10 **Jiazu Zhou**, Isoperimetric problems and Alexandrov-Fenchel inequalities

6.28

Tuesday morning

8:30-9:30 **Xiaodong Wang**, Comparison and rigidity results on compact Riemannian manifolds with boundary

9:40-10:40 **Richard Bamler**, Ricci flows with bounded scalar curvature

11:00-12:00 **Jeff Streets**, Generalized Kahler-Ricci flow

Tuesday afternoon

2:00-3:00 **Rivière Tristan**, The Variations of Yang-Mills Lagrangian

3:10-4:10 **Claudio Arezzo**, Kahler constant scalar curvature metrics on blow ups and resolutions of singularities

4:30-5:30 **Haotian Wu**, Asymptotic shapes of neckpinch singularities in geometric flows

6.29

Wednesday morning

8:30-9:30 **Xiaodong Wang**, Comparison and rigidity results on compact Riemannian manifolds with boundary

9:40-10:40 **Richard Bamler**, Ricci flows with bounded scalar curvature

11:00-12:00 **Jeff Streets**, Generalized Kahler-Ricci flow

Wednesday afternoon

2:00-3:00 **Xianzhe Dai**, Index theory and its geometric applications

3:10-4:10 **Xianzhe Dai**, Index theory and its geometric applications

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Schedule and abstracts

6.30

Thursday morning

8:30-9:30 **Yannik Sire**, Some bounds on eigenvalues of the Laplace-Beltrami operator on Riemannian manifolds

9:40-10:40 **Rivière Tristan**, The Variations of Yang-Mills Lagrangian

11:00-12:00 **Xianzhe Dai**, Index theory and its geometric applications

Thursday afternoon

2:00-3:00 **Yannik Sire**, Some bounds on eigenvalues of the Laplace-Beltrami operator on Riemannian manifolds

3:10-4:10 **Yannik Sire**, Some bounds on eigenvalues of the Laplace-Beltrami operator on Riemannian manifolds

7.1

Friday morning

9:00-9:50 **Tobias Lamm**, Conformal Willmore tori in \R^4“

10:00-10:50 **Hao Fang**, Volume bounds for conic 2-spheres with curvature bounds

11:10:-12:00 **Lei Zhang**, Local Mass Concentration and A priori estimate for singular Toda systems of Rank 2

Friday afternoon

2:00-2:50 **Rivière Tristan**, Willmore Minmax Surfaces and the Cost of the Sphere Eversion

3:00-3:50 **Zhou Zhang**, Mean Curvature Flow over Almost Fuchsian Manifolds

4:10-5:00 **Chao Qian**, Isoparametric foliation and its applications on related geometric problems

5:10-5:40 **Xishen Jin**, Twisted and conical K\"ahler-Ricci soliton on Fano manifold

7.2

Saturday morning

9:00-9:50 **Takashi Shioya**, High-dimensional spaces in metric measure geometry

10:00-10:50 **Gang Liu**, Gromov-Hausdorff limit of Kahler manifolds with bisectional curvature lower bound

11:10:-12:00 **Lu Wang**, Hypersurfaces of Low Entropy

Saturday afternoon

2:00-2:50 **Qi Zhang**, New Volume Comparison results and Applications to degeneration of Riemannian metrics

3:00-3:50 **Liangming Shen**, Conical Kahler-Einstein metrics along a simple normal crossing divisor

4:10-5:00 **Tamas Darvas**, Metric geometry on the space of Kahler metrics

7.3

Sunday morning

9:00-9:50 **Fuquan Fang**, Reflections in Riemannian geometry

10:00-10:50 **Yuxiang Li**, Willmore minimizers when prescribed isoperimetric ratio goes to 0

11:10:-12:00

Sunday afternoon

2:00-2:50 **Yann Bernard**, Ends of immersed minimal and Willmore surfaces in asymptotically flat spaces

3:00-3:50 **Xi Zhang**, The Hermitian-Yang-Mills flow on reflexive Higgs sheaves

4:10-5:00 **Chengjie Yu**, Estimate of Higher Steklov Eigenvalues

5:10-5:40 **Wenshuai Jiang**, L^2 curvature bounds on manifolds with bounded Ricci curvature

We will provide lunch boxes after morning talks.

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Schedule and abstracts

Title and Abstract:

Xiaodong Wang（Michigan State University)

Title: *Comparison and rigidity results on compact Riemannian manifolds with boundary*

Abstract: For compact Riemannian manifolds with nonempty boundary, it is interesting to study the relationship between the geometry on the boundary and geometry of the interior. I will discuss comparison and rigidity results for manifolds with a lower bound for the Ricci curvature. The focus will be on sharp geometric inequalities that yield rigidity results in the equality case.

If time allows I will also discuss results for manifolds with a lower bound for the scalar curvature. Many results and questions in this direction are motivated by general relativity.

Richard Bamler (Berkeley)

Title: *Ricci flows with bounded scalar curvature*

Abstract: We discuss recent results on the structure theory of Ricci flows with bounded scalar curvature. Such flows naturally occur in the study of Kaehler Ricci flows on Fano manifolds. The Hamilton-Tian Conjecture states that Kaehler Ricci flows on Fano manifolds converge, away from a subset of codimension larger or equal to 4. The structure theory presented in this mini course is purely Riemannian and it gives an affirmative answer to a (much more general) Riemannian version of the Hamilton-Tian Conjecture.

Jeff Streets (UC Irvine)

Title: *Generalized Kahler-Ricci flow*

Abstract: In this minicourse I will introduce geometric and analytic aspects of the generalized Kahler-Ricci flow. Generalized Kahler geometry was originally discovered through investigations into supersymmetry, and was later rediscovered in a purely geometric context in work of Hitchin/Gualtieri. The generalized Kahler-Ricci flow was introduced in joint work of myself and G. Tian and is a natural tool for addressing questions of existence and moduli of canonical metrics and the topology of generalized Kahler manifolds. We will begin with a brief introduction to generalized Kahler geometry, and then introduce fundamental aspects of the equation. In the final lecture we will prove global existence and convergence results, and what open problems remain.

Rivière Tristan (ETH Zurich)

Title: *The Variations of Yang-Mills Lagrangian*

Abstract: Yang-Mills theory is growing at the interface between high energy physics and mathematics.
It is well known that Yang-Mills theory and Gauge theory in general had a profound impact on the development of modern differential and algebraic geometry. One could quote Donaldson invariants in four dimensional differential topology, Hitchin Kobayashi conjecture relating holomorphic bundles over Kähler manifolds and Mumford stability in complex geometry or also Gromov Witten invariants in symplectic geometry...etc. While the influence of Gauge theory in geometry is quite notorious, one tends sometimes to forget that Yang-Mills theory has been also

at the origin of fundamental progresses in the non-linear analysis of Partial Differential Equations in the last decades.

The purpose of this mini-course is to present the variations of this important lagrangian. We shall raise analysis question such as existence and regularity of Yang-Mills minimizers or critical point of Yang-mills lagrangian in general.

We will first describe during the first half of the course the progresses which have been made in these directions for the critical dimension 4 and below. At this occasion we will mostly insist on the important contributions by K.Uhlenbeck from the late seventies - early eighties. The second part of the mini-course will be devoted to the study of Yang-Mills fields in dimension larger than 4. We will first describe the Lin-Tian concentration compactness method in super critical dimension as well as the epsilon-regularity results obtained by Tao-Tian and Meyer-Rivière. In the last part of the course we shall present very recent results obtained in collaboration with M. Petrache regarding the ad-hoc framework for producing Yang-Mills critical points in dimension larger than 4.

Jiazu Zhou（Southwest University）

Title: *Isoperimetric problems and Alexandrov-Fenchel inequalities*

Abstract: The classical isoperimetric problem is to determine a plane figure of the largest possible area whose boundary has a specified length. The classical isoperimetric problem was known in the Ancient Greece. However, the first mathematically rigorous proof was obtained only in the 19th century. The classical Brunn-Minkowski theory, also known as the theory of mixed volumes, originated with Minkowski when he combined his concept of mixed volume with the Brunn-Minkowski inequality. One of Minkowski's major contributions to the theory was to show how his theory could be developed from a few basic concepts, such as support function, vector addition, and volume.

The isoperimetric problem was characterized by isoperimetric inequality. The natural extensions are Minkowski inequalities and Alexandrov-Fenchel inequalities for mixed volumes. Minkowski inequality in a normed space is equivalent to Sobolev inequality. The past investigation found that those known geometric inequalities are equivalent to some analytic inequalities. The latest research focus on finding new (Bonnesen-style) isoperimetric imequalities and Alexandrov-Fenchel inequalities that have analogues in differential geometry, complex geometry, algebraic geometry and convex geometric analysis.

Recent progresses towards an Orlicz-Brunn-Minkowski theory was made by Lutwak-Yang-Zhang (Adv. Math. 223 (2010) 220-242, J. Diff. Geom. 84 (2010) 365 - 387) and Ludwig-Reitzner (Adv. Math.
224 (2010) 2346-2360, Ann. of Math. 172 (2010) 1223-1271) for valuation, and the dual Orlicz-Brunn-Minkowski theory was made by Xu-Zhou-Zhu (Adv.Math. 264(2014) 700-725) and Gardner-Hug-Weil-Ye (JMAA, 430 (2015), no. 2, 810-829).

Haotian Wu (University of Oregon)

Title: *Asymptotic shapes of neckpinch singularities in geometric flows*

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Abstract: Geometric flows such as Ricci flow and mean curvature flow are natural and important tools to understand the geometry and topology of Riemannian manifolds. Geometric flows are nonlinear parabolic (heat) partial differential equations that tend to develop singularities in finite time. A useful approach to analyzing the singularities is the technique of matched asymptotics, which can provide detailed and precise information including the rates of curvature blow-up, the set of points where a singularity forms, and the behavior of the solution in a space-time neighborhood of that singularity. In this talk, we will survey the results concerning the asymptotic shapes of neckpinch singularities in Ricci flow and mean curvature flow.

Xianzhe Dai (UCSB)

Title: *Index theory and its geometric applications*

Abstract: The Atyiah-Singer index theory is one of the landmark results in 20th century mathematics, unifying several great theorems in differential geometry, algebraic geometry, and differential topology. In the first lecture, we will review the Dirac operator, characteristic classes, the Atiyah-Singer index theorem, as well as the Atiyah-Patodi-Singer index theorem. The remaining two lectures will concentrate on the geometric applications. The second lecture will concern the stability issues of Einstein metrics, and the third one the construction of interesting geometric invariants which can distinguish various geometric structures.

Yannik Sire (Johns Hopkins University)

Title: *Some bounds on eigenvalues of the Laplace-Beltrami operator on Riemannian manifolds*

Abstract: I will describe several recent results where one explores the relationship between bounds on eigenvalues of the Laplace-Beltrami operator on compact manifolds and minimal sub manifolds in the sphere.
During the first lecture, I will give universal bounds on eigenvalues properly renormalized by the area of the surface and provide a theorem of existence for extremal metrics on conformal classes. During the second lecture, I will explore the relationship between critical metrics and minimal sub manifolds on the sphere. Finally, the third lecture will be devoted to bounds on the number of negative eigenvalues for Schrodinger operators.

Tobias Lamm（Karlsruhe Institute of Technology）

Title: *Conformal Willmore tori in \R^4"*

Abstract: In this talk I am going to present recent existence and non-existence results for conformal Willmore Tori in $\mathbb{R}^4$ which were obtained in a collaboration with Reiner M. Schätzle (Tübingen).

Hao　Fang（University of Iowa）

Title: *Volume bounds for conic 2-spheres with curvature bounds*

Abstract: We discuss the best volume bounds for conic 2-spheres with curvature bounds. In particular, we compute the Gromov volume for conic 2-spheres. We show that extremal metrics have rotational symmetry. Also, we will discuss some technical details dealing with negative curvature case. This is a joint work with Mijia Lai.

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Lei Zhang（University of Florida）

Title: *Local Mass Concentration and A priori estimate for singular Toda systems of Rank 2*

Abstract: A Toda system is a nonlinear second order elliptic system with exponential nonlinearity. It is very commonly observed in physics and has many ties with algebraic geometry. From analytic viewpoints it is challenging since the solutions do not have symmetry, maximum principles cannot be applied and the structures of global solutions are incredibly complicated. In this joint work with Chang-shou Lin and Juncheng Wei, we use a unified approach to discuss all rank two singular Toda systems. First for local systems we prove that all weak limits of mass concentration belong to a very small finite set. Then for systems defined on compact Riemann surface we establish a priori estimate. Our approach is a combination of delicate blowup analysis and fundamental tools from algebraic geometry.

Rivière Tristan (ETH Zurich)

Title: *Willmore Minmax Surfaces and the Cost of the Sphere Eversion*

Abstract: We develop a general Minmax procedure in Euclidian spaces for constructing Willmore surfaces of non zero indices. We implement this procedure to the Willmore Minmax Sphere Eversion in the 3 dimensional euclidian space in order to compute the cost of this famous eversion.

Zhou Zhang（The University of Sydney）

Title: *Mean Curvature Flow over Almost Fuchsian Manifolds*

Abstract: We consider mean curvature flow in the non-Euclidean setting. More specifically, the ambient 3-manifold is almost Fuchsian, which is hyperbolic and of great interest in low-dimension topology and differential geometry. The (closed) surface of evolution under study is a graph. After some general discussion, we restrict ourselves to the case of Fuchsian manifold. Under a mild assumption on the graph property of the initial surface, we can justify the smooth convergence to the minimal surface at time infinity. The discussion could also be helpful in constructing examples of finite time singularities. This is a joint work with Zheng Huang (CUNY) and Longzhi Lin(UCSC).

Chao Qian（Beijing Institute of Technology）

Title: *Isoparametric foliation and its applications on related geometric problems*

Abstract: In this talk, we will discuss two sequences of minimal isoparametric hypersurfaces, constructed via representations of Clifford algebras. Based on these, we give estimates on eigenvalues of the Laplacian of the focal submanifolds of isoparametric hypersurfaces in unit spheres.

Eells and Lemaire [EL83] posed a problem to characterize the compact Riemannian manifold M for which there is an eigenmap from M to S^n. As another application of our constructions, the focal maps give rise to many examples of eigenmaps from minimal isoparametric hypersurfaces to unit spheres.

Most importantly, by investigating the second fundamental forms of focal submanifolds of isoparametric hypersurfaces in unit spheres, we provide infinitely many counterexamples to two conjectures of Leung [Le91] (posed in 1991) on minimal submanifolds in unit spheres.

Notice that these conjectures of Leung have been proved in the case that the normal connection is flat.
This talk is based on the joint work with Prof. Zizhou Tang.

Takashi Shioya (Tohoku University)

Title: *High-dimensional spaces in metric measure geometry*

Abstract: Gromov introduced a new topology on the space of metric measure spaces, which is weaker than the measured Gromov-Hausdorff topology and is based on concentration of measure phenomenon.
In this talk, we show some of our works on convergence of spaces with unbounded dimension in Gromov's topology.

Gang Liu（University of California, Berkeley）

Title: *Gromov-Hausdorff limit of Kahler manifolds with bisectional curvature lower bound*

Abstract: We prove that the Gromov-Hausdorff limit of Kahler manifolds with bisectional curvature lower bound and noncollapsed volume is homeomorphic to a normal complex analytic space. As a consequence, if M is a complete noncompact Kahler surface with positive bisectional curvature and noncollapsed volume, then it is simply connected.

Lu Wang (University of Wisconsin-Madison)

Title: *Hypersurfaces of Low Entropy*

Abstract: The entropy is a natural geometric functional introduced by Colding-Minicozzi to study the singularities of mean curvature flow, and it roughly measures the complexity of a hypersurface of Euclidean space. In this talk, I will survey some recent progress with Jacob Bernstein on understanding the geometry and topology of hypersurfaces with low entropy.

Qi Zhang (UC Riverside)

Title: *New Volume Comparison results and Applications to degeneration of Riemannian metrics.*

Abstract: We consider a condition on the Ricci curvature involving vector fields, which is broader than the Bakry-\'Emery Ricci condition. Under this condition a volume comparison, Laplacian comparison, isoperimetric inequality and gradient bounds are proven on the manifold.
Specializing to the Bakry-\'Emery Ricci curvature condition, we initiate an approach to work on the original manifold, which yields, under a weaker than usual assumption, the results mentioned above for the original manifold. These results are different from well known ones in the literature where the conclusions are made on the weighted manifold instead.
Applications on convergence and degeneration of Riemannian metrics under this curvature condition are given. To this effect, the gradient of the potential function is allowed to have singularity of order close to 1 while the traditional method of weighted manifolds allows bounded gradients. The result also covers general solitons rather than just gradient solitons.

This is a joint work with Zhu, Meng.

Liangming Shen（Univ. British Columbia）

Title: *Conical Kahler-Einstein metrics along a simple normal crossing divisor*

Abstract: On a Fano Kahler-Einstein manifold with a simple normal crossing divisor which is proportional to c_{1}(M), if the divisor satisfies certain conditions, we prove that there exist conical Kahler-Einstein metrics with prescribed cone angles along the divisor. If time permits, we will introduce a new curvature estimate for conic metrics along one simple normal crossing divisor.

Tamas Darvas （University of Maryland）

Title: *Metric geometry on the space of Kahler metrics*

Abstract: We introduce the L^p-Mabuchi Finsler structure on the space of Kahler metrics. We argue that the completion of the associated path length metric structures are well known objects from finite energy pluripotential theory. Time permitting we will mention how the use of the arising metric geometry is useful in settling questions about the long time behavior of the Calabi flow and J-properness.

Xishen Jin (University of Science and Technology of China)

Title: *Twisted and conical K\"ahler-Ricci soliton on Fano manifold*

Abstract: In this talk, we will show the relation between properness of energy functional and existence of twisted K\"ahler-Ricci soliton. Furthermore, the existence of conical K\"ahler-Ricci soliton will be considered. In particular, under some assumptions, we get the properness of modified log K-energy and the existence of conical K\"ahler-Ricci soliton with suitable cone angle. This work is jointed with Jiawei Liu and Xi Zhang.

Fuquan Fang (Capital Normal University)

Title: *Reflections in Riemannian geometry*

Abstract: We provide an equivariant description / classification of all com-plete (compact or not) nonnegatively curved manifolds M together with a co-compact action by a reflection group W, andmoreover, classify such W. In particular, we show that the building blocks consist of the classical constant curvature models and generalized open books with nonnegatively curved bundle pages, and derive a corresponding splitting theorem for the universal cover. This is a joint work with Karsten Grove.

Yuxiang Li（Tsing Hua Univercity）

Title: *Willmore minimizers when prescribed isoperimetric ratio goes to 0*

Abstract: Let $f_\sigma:S^2\rightarrow R^3$ be embedding which attains $$\beta_\sigma=\inf_{\mu(f)=1,V(f)=\sigma}W(f).$$ It was proved by Schygulla that the image of f_σ converges to a double sphere in the sense of varifold as $\sigma\rightarrow 0$.

In this talk, we will show that f_σ converges to a double sphere and a catenoid in the bubble tree sense. This is a joint work with E. Kuwert.

Yann Bernard (Monash U, Australia)

Title: *Ends of immersed minimal and Willmore surfaces in asymptotically flat spaces*

Abstract: We study ends of an oriented, immersed, non-compact, complete Willmore surfaces, which are critical points of the integral of the square of the mean curvature, in asymptotically flat spaces of any dimension; assuming the surface has L^2-bounded second fundamental form and satisfies a weak power growth on the area. We give the precise asymptotic behavior of an end of such a surface. This asymptotic information is very much dependent on the way the ambient metric decays to the Euclidean one. Our results apply in particular to minimal surfaces. This is joint-work with Tristan Rivière.

Xi Zhang（University of Science and Technology of China）

Title: *The Hermitian-Yang-Mills flow on reflexive Higgs sheaves*

Abstract: In this talk, we consider the asymptotic behavior of the Hermitian-Yang-Mills flow on reflexive Higgs sheaves. We prove that if a reflexive Higgs sheaf is not stable, then it breaks up into a direct sum of Hermitian-Einstein reflexive Higgs sheaves via the Hermitian-Yang-Mills flow. Furthermore, we show that the direct sum is isomorphic to the double dual of the graded Higgs sheaves associate to Harder-Narasimhan-Seshadri filtration. This work is joint with JiaYu Li and ChuanJing Zhang.

Chengjie Yu （Shantou Univeristy）

Title: *Estimate of Higher Steklov Eigenvalues*

Abstract: In this talk, we will first give a brief survey of the estimates of Steklov eigenvalue. Then, some new estimates for higher Steklov eigenvalues will be discussed.

Wenshuai Jiang (Peking University)

Title: *L^2 curvature bounds on manifolds with bounded Ricci curvature*

Abstract: In this talk, we will discuss the L^2 curvature estimates on manifolds with bounded Ricci curvature and noncollapsing volume. Firstly, we will talk about the background, then we introduce the concept of neck region which appears everywhere in our proof. After that, we would sketch the whole proofs. At last, we would focus on the technical details and some new observations on neck region. This is joint work with Prof. Aaron Naber.