Summer School on Complex Analysis: Interplay of Dynamics, Geometry, and Probability

September 3 - September 12, 2026

Room 77201, Jingchunyuan 78, BICMR

Organizing Committee:

Hanlong Fang (Peking University)

Zhiqiang Li (Peking University)

Speakers:

Kari Astala (University of Helsinki)

Mario Bonk (UCLA)

Sylvester Eriksson-Bique (University of Jyväskylä)

Ido Grayevsky (University of Bristol)

Kostya Khanin (BIMSA)

Sam Mellick (BICMR)

Pekka Pankka (University of Helsinki)

	3-Sep-26 Thursday	4-Sep-26 Friday	5-Sep-26 Saturday	6-Sep-26 Sunday	7-Sep-26 Monday	8-Sep-26 Tuesday	9-Sep-26 Wednesday	10-Sep-26 Thursday	11-Sep-26 Friday	12-Sep-26 Saturday
9:00 - 10:00			Bonk	Khanin						Astala
10:00 - 11:00			Pankka	Khanin						Mellick
11:00 - 12:00			Eriksson-Bique	Eriksson-Bique						Bonk
12:00 - 13:00										
13:00 - 14:00										
14:00 - 15:00			Khanin	Pankka					Khanin (SMS colloquium)	Grayevsky
15:00 - 16:00	Bonk (BICMR colloquium)		Khanin	Eriksson-Bique						Mellick
16:00 - 17:00		Pankka (SMS colloquium)	Grayevsky	Khanin		Astala (BICMR colloquium)				
17:00 - 18:00										
18:00 - 19:00										
19:00 - 20:00					Pankka		Grayevsky	Astala		
20:00 - 21:00					Eriksson-Bique		Bonk	Bonk		
21:00 - 22:00					Bonk		Pankka	Grayevsky		

Summer School Title and Abstract

Titles and abstracts of colloquium talks will announce separately.

Mario Bonk (UCLA)
5 lectures
Title: An introduction to Sullivan's dictionary

Abstract:
In 1985 Dennis Sullivan pointed out some similarities between concepts, statements, and methods in the investigations of the dynamics of Kleinian groups and of rational functions. These analogies are now known as *Sullivan's dictionary* and have been a fruitful source of inspiration for research. In my lectures I will give an introduction to this area.

Sylvester Eriksson-Bique (University of Jyväskylä)
4 lectures
Title: Conformal dimension and energies for fractals

Abstract:
Conformal dimension and its attainment problem are a way of approaching Cannon's conjecture. This approach faces a significant difficulty: both are difficult to understand for even the simplest spaces - let alone boundaries of groups required for Cannon. In these lectures, we will first present the main classical tools for their study: combinatorial modulus, Loewner spaces and modulus. We will explain how these characterize minimality. Then, we move on to the problem of attainment, and introduce a new tool that has arisen in the last few years: energy measures and non-linear energy forms. Through this leap, we will work through the recent work of myself and Anttila to show that many product spaces fail to attain their conformal dimension. This will involve introducing the important tools of energy dominant measures, the singularity phenomenon and blow-ups. This will solve a conjecture of Kleiner and reveals a new way how attainment can fail. It will be helpful to know about metric spaces, functional analysis, measure theory and some geometric measure theory, but I will try to be largely self-contained. You will get more of Lecutre 4, if you have looked at the notion of Gromov-Hausdorff convergence.

Lecture 1: **Modulus.** Introduction and modulus as a tool for conformal dimension. Loewner spaces, and the result of Carrasco-Piaggio and Keith-Laakso.
Lecture 2: **Energies.** How to find a measure? A leap to energies. Constructing energies on products. The result of Kigami on existence of energies, and comparability of modulus and energy.
Lecture 3: **Energy dominant measures** and their behavior on Loewner spaces and product spaces. Combinatorially Loewner spaces and characterization of conformal dimension.

Lecture 4: **Singularity phenomenon and blow-up analysis.** Adapt a method of Cheeger to get singularity, and use blow-up analysis to solve Kleiner's conjecture by proving that product of Sierpiński carpets do not attain its conformal dimension.

Pekka Pankka (University of Helsinki)
4 lectures
Title: Uniformly quasiregular dynamics in disguise

Abstract:
Holomorphic dynamics has a higher dimensional extension in the form uniformly quasiregular mappings: we consider self-maps of Riemannian manifolds for which the distortion of the mapping stays bounded under iteration. After preliminaries on uniformly quasiregular mappings, I will discuss a recent work with Zhiqiang Li and Hanyun Zheng on a class of branched covering mappings, called Thurston-type maps. These topological mappings carry intrinsic (visual) metrics, which we may uniformize in terms of uniformly quasiregular dynamics.

Ido Grayevsky (University of Bristol)
4 lectures
Title: coarse geometry of Lie groups

Abstract:
Studying the coarse geometry of groups has proven to be a very useful tool over the past 50 years or so. These talks will focus on the class of Lie group. Semisimple Lie groups have been classified upto quasiisometry (QI), while the problem of QI classification of solvable Lie groups is a wide open problem. In this context, in recent years there has been much progress in the study of sublinear bilipschitz equivalence (SBE) - maps that are well adapted to study asymptotic properties of solvable Lie groups.

We will present the history of the QI classification of semisimple Lie groups, introducing the key ideas that arise in the different proofs. We will then discuss the SBE classification of semisimple Lie groups, highlighting the differences between the boundary objects that are used in rank 1 versus high rank groups. Finally, we will consider SBE in the context of solvable Lie groups and show how it can contribute to the problem of QI classification of solvable Lie groups.

Lecture 1: QI-classification of semisimple Lie groups: Visual boundaries and asymptotic cones, Boundary maps and flat rigidity.

Lecture 2: SBE in Rank 1: sublinear Morse, sublinear quasi-conformal maps.

Lecture 3: SBE in high rank: lattices, truncated symmetric space, and an arithmeticity criterion.

Lecture 4: SBE in solvable Lie groups with a view towards the QI classification: A product decomposition theorem and some concrete examples.

Kostya Khanin (BIMSA)
5 lectures
Title: On number theory, termodynamic formalism and renormalization
Abstract:
In this mini course I plan to introduce some constructions related to the thermodynamic formalism and dynamical renormalization. I will start with the continued fractions algorithm which will serve as a motivation for further discussions.

Kari Astala (University of Helsinki)
2 lectures
Title: Geometry of random tilings via complex structures and the Beltrami equation.

Abstract:
Under suitable boundary conditions, scaling limits of random tilings (the dimer models) present surprising geometric features: one observes both deterministic parts and disordered parts (or frozen and liquid parts) with interesting geometric properties which one wants to understand.

In this course we first explain briefly how the combinatorics of random tilings can be understood via discrete complex analysis, and then how in the scaling limit this leads special degenerate Beltrami differential equations and the complex structures they provide.

With these tools, it is possible to understand the geometry of the boundary between the frozen/deterministic and liquid/random phases for all dimer models. It turns out that in this class (and even beyond) the geometry of the frozen boundaries is universal, i.e. independent of the specific model. Another goal of the course is to describe and understand the special quasiconformal mappings these Beltrami equations lead to.

Sam Mellick (BICMR)
2 lectures
Title: Cost, fixed price, and rank gradient: recent developments

Abstract:
Cost is a central invariant in measured group theory. It is the dynamical / ergodic theoretic generalisation of the notion of rank, the minimum size of a generating set for a group. Cost is a natural measure of the complexity of probability measure preserving (pmp) actions, and it has applications in numerous fields such as percolation theory, asymptotic group theory, the study of paradoxical decompositions, and operator algebra theory. A group is said to have fixed price if all of its essentially free pmp actions have the same cost. A major open problem is to exhibit a group which does not have this property. The fixed price property itself has many applications, such as vanishing of rank gradient (showing that the rank of a lattices grows sublinearly in their covolume).

A recent breakthrough was the proof of the fixed price property for higher rank semisimple Lie groups. This made use of an interesting and natural stochastic geometric object, the ideal Poisson Voronoi

tessellation. Numerous works have appeared subsequent to this, drawing inspiration from it. In these talks, I will explain cost and survey these recent developments and simplifications. No prior familiarity with cost is assumed.