INTRODUCTION TO TALKS AT THE DENG-HANI-MA READING SEMINAR

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In previous talks of this seminar series, we have systematically developed the cumulant expansion framework. To proceed, it now suffices to bound $\|E_H\|_{L^1}$, and my part will be dedicated to developing such estimates. Our analysis proceeds in several key steps. This is detailed in Sections 7–9 of the celebrated paper by Deng, Hani, and Ma (Version 3).

Step 1: Reduction to $\mathcal{I}_{\mathbb{M}}(Q_{\mathbb{M}})$ Estimates. We begin by reducing the estimate for $\|E_H\|_{L^1}$ to individual bounds for a certain quantity $\mathcal{I}_{\mathbb{M}}(Q_{\mathbb{M}})$ given by local integral expressions, which is equivalent to $\left\||\mathcal{I}\mathcal{N}_{\mathbb{M}}|\right\|_{L^1}$. This involves two main substeps:

- **Counting the number of molecules**: Establish the upper bound for the number of molecules.
- **Local Representation Formula**: Rewrite the integrals $\left\||\mathcal{I}\mathcal{N}_{\mathbb{M}}|\right\|_{L^1}$ in $\|E_H\|_{L^1}$ as the integrals $\mathcal{I}_{\mathbb{M}}(Q_{\mathbb{M}})$ involving Dirac functions that localize each collision and overlap.

Step 2: Operations on molecules. To estimate $\mathcal{I}_{\mathbb{M}}(Q_{\mathbb{M}})$, we introduce a **operation sequence** that reduces the molecule $\mathbb{M}$ to smaller and simpler **elementary molecules**. This involves:

- **Cutting Operations and Elementary Molecules**: We will explain the operations, including cutting, deleting and splitting. We will study the behavior of $\mathcal{I}_{\mathbb{M}}(Q_{\mathbb{M}})$ under these operations.
- **Estimates for Elementary Molecules**: We will derive precise bounds for each elementary molecule and classify these molecules based on their contribution to the integration.

Step 3: Final bound for $\|E_H\|_{L^1}$. Assuming a suitably chosen operation sequence, we establish the desired bound for $\|E_H\|_{L^1}$.

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