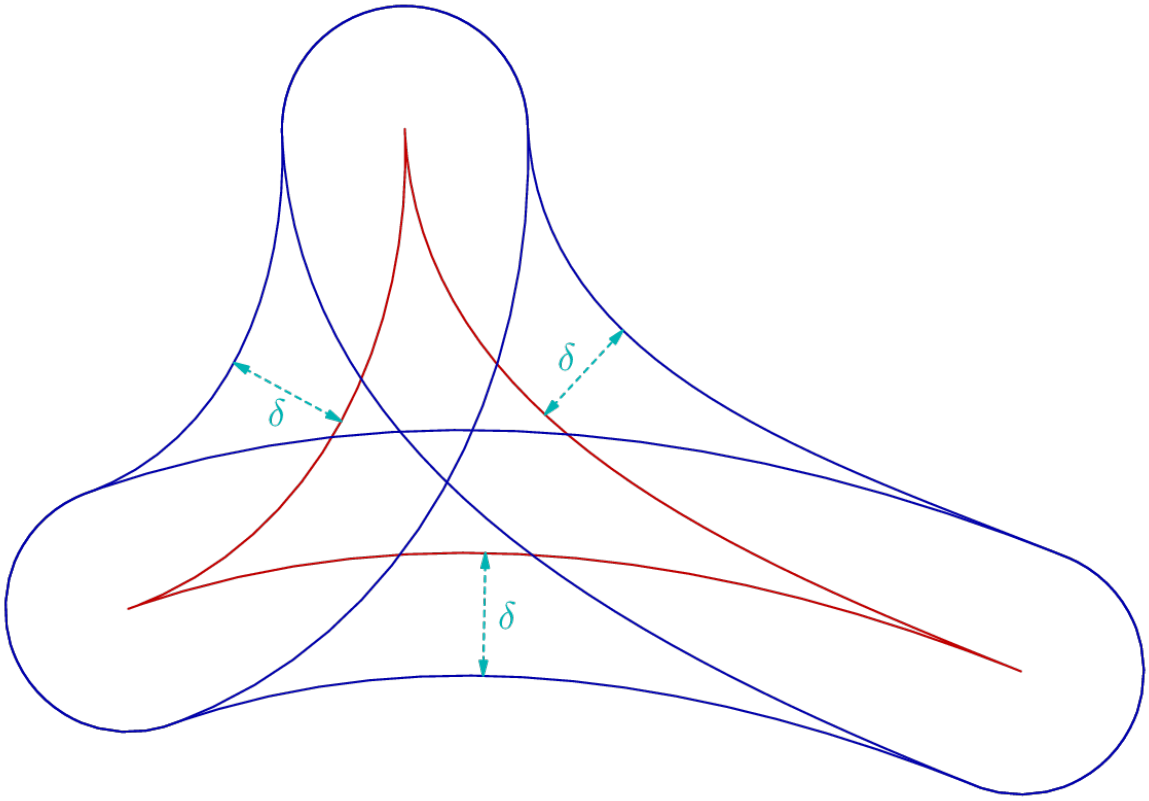
Summer School on Geometric Group Theory

Peking University, Beijing, China
June 9-13, 2025



Supported by
Beijing International Center for Mathematical Research (BICMR)

Lecturers

Mark Hagen (University of Bristol)

Vadim Kaimanovich (University of Ottawa)

Anders Karlsson (Université de Genève)

Alessandro Sisto (Heriot-Watt University)

Abdul Zalloum (Harbin Institute of Technology)

Conference venue:

Lecture Room, Jiayibing Building, Jingchunyuan 82, BICMR
(镜春园 82 号甲乙丙楼报告厅)

Live streaming:

VooV Meeting (腾讯会议)：962-9315-6070

Meeting Password: 202569

Organizing Committee

Yi Liu (Peking University)

Wenyuan Yang (Peking University)

Mini-courses Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00-09:30	Registration				
09:30-10:30	V. Kaimanovich (1)	A. Sisto (1)	A. Zalloum (4)	A. Karlsson (1)	M. Hagen (3)
10:30-11:00	Tea break				
11:00-12:00	A. Zalloum (1)	V. Kaimanovich (3)	M. Hagen (1)	A. Sisto (3)	A. Karlsson (3)
12:00-14:00	Lunch break				
14:00-15:00	V. Kaimanovich (2)	A. Sisto (2)	V. Kaimanovich (4)	A. Karlsson (2)	M. Hagen (4)
15:00-15:30	Tea break				
15:30-16:30	A. Zalloum (2)	A. Zalloum (3)	M. Hagen (2)	A. Sisto (4)	A. Karlsson (4)
16:30 onwards	Free discussion				

Mini-courses

Real cubings and their applications

Mark Hagen
University of Bristol

I will start with a short introduction to median metric spaces (a common generalisation of CAT(0) cube complexes and real trees, among other examples), and also define coarse median spaces as defined by Bowditch. Next, I will recall the class of hierarchically hyperbolic spaces (HHS), which are a subclass of coarse median spaces. There is a fine-geometric analogue: if one imposes some additional conditions on a median space — analogous to the conditions on a coarse median space needed to make an HHS — one gets the notion of a real cubing; these will be the main objects in the course.

After defining them, the goal will be explaining two things: (1) a sketch of the proof that any asymptotic cone of an HHS is bilipschitz equivalent to a real cubing, and (2) why the extra structure in a real cubing, beyond just the median structure, is desirable.

The minicourse will use results from many authors; the notion of a real cubing comes from joint work with M. Casals-Ruiz and I. Kazachkov, a draft of which is accessible here: https://www.wescac.net/cones_july_2024-public.pdf. In terms of that document, the course will cover a subset of Sections 4-7, 12-17, and a sketch of the results in Part 4.

Boundary measures on hyperbolic groups

Vadim Kaimanovich
University of Ottawa

The presence of a rich topological boundary is an indispensable feature of hyperbolic groups. The purpose of this mini-course is to give an introduction to the approach based on considering various natural measures or measure classes on the hyperbolic boundary.

I will begin with an overview of the Rokhlin theory of standard measure spaces, and will further look at the basic properties and interrelations of boundary measures arising from various dynamical considerations: Hausdorff and conformal measures, geodesic currents, and harmonic measures of random walks.

Metric functionals and their applications

Anders Karlsson

Université de Genève

This mini-course will develop a part of metric geometry in analogy with functional analysis. Like the linear theory there is the important concept of functionals (Busemann and horofunctions) and spectral theorems (for nonexpansive maps and an ergodic theorem for random products of maps. Isometries moreover enjoy a fixed point theorem). Examples and applications include finitely generated groups, invertible linear operators, holomorphic maps, surface homeomorphisms, certain neural networks, and reinforcement learning.

References:

1. Gouëzel, S; Karlsson, A. Subadditive and multiplicative ergodic theorems, J. Eur. Math. Soc. 22 (2020), no. 6, 1893–1915.

2. Karlsson, A. From linear to metric functional analysis. Proc. Natl. Acad. Sci. USA 118 (2021), no. 28, Paper No. e2107069118, 5 pp.

3. Karlsson, A. A metric fixed point theorem and some of its applications. Geom. Funct. Anal.34(2024), no.2, 486–511.

Introduction to hierarchical hyperbolicity

Alessandro Sisto

Heriot-Watt University

Hierarchically hyperbolic spaces were introduced about ten years ago as a common framework to understand the geometry of mapping class groups and CAT(0) cube complexes, and since then many other examples, as well as applications, have been found. I will give an introduction to this notion, emphasising connections with cubical geometry, specifically via coarse medians and the cubical approximation theorem (both relevant for the other two mini-courses).

Main source: What is a hierarchically hyperbolic space? (arXiv: 1707.00053).

Metric structures from walls and applications to mapping class groups

Abdul Zalloum
Harbin Institute of Technology

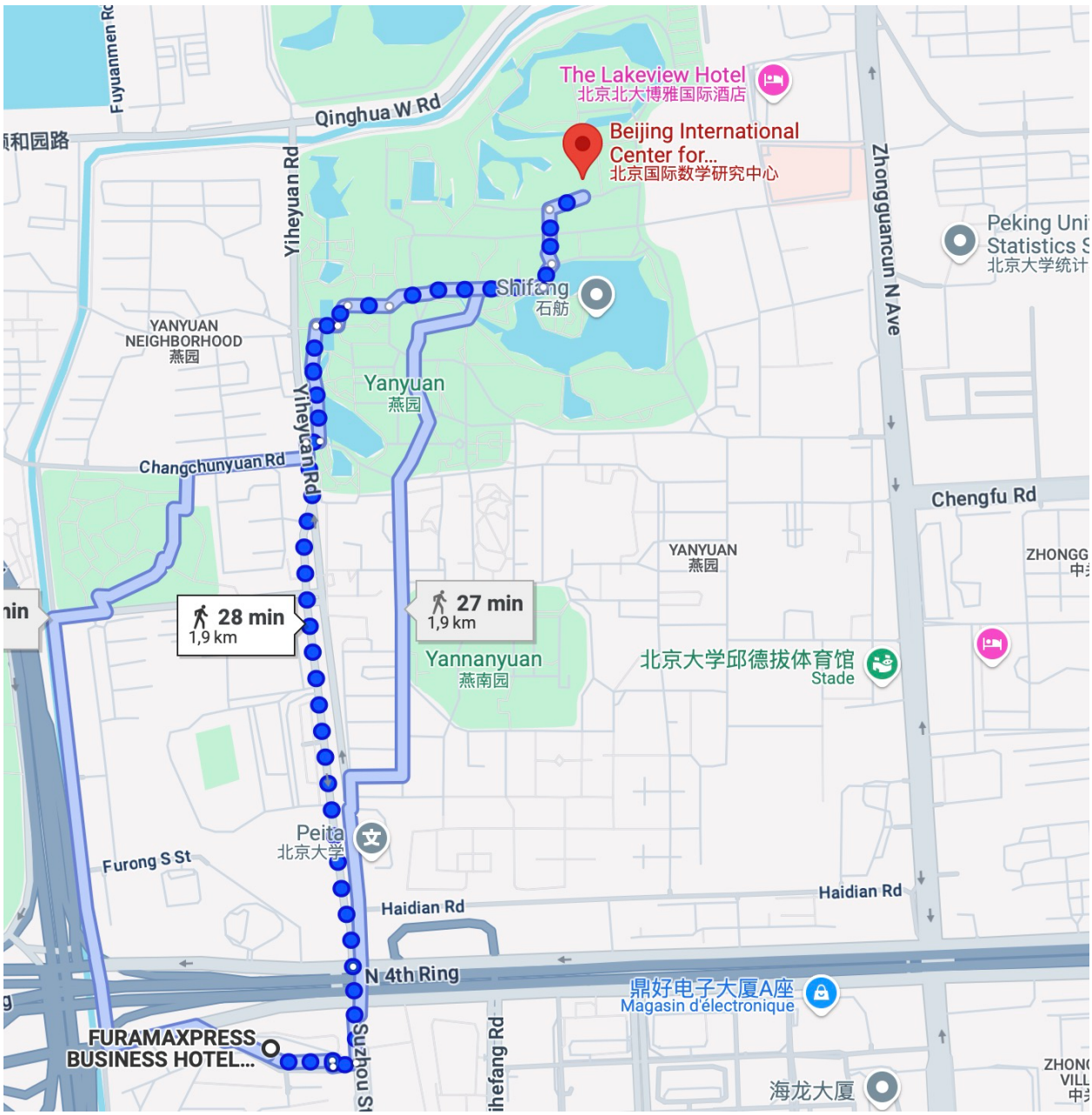
Sageev's celebrated construction shows that, starting with a set S and a collection of bipartitions W satisfying a finiteness condition, one can construct a CAT(0) cube complex. This course will discuss a recent generalization of this idea: given a set S and a collection of walls W, one can build actions on various types of metric spaces, including hyperbolic and injective metric spaces. I will then present several applications of this construction, including: constructing geometric actions of mapping class groups on injective spaces; establishing an equivariant quasi-isometric embedding of the curve complex into finite products of quasi-trees; exhibiting globally stable cylinders for residually finite hyperbolic groups; and constructing a universal hyperbolic space on which all strongly contracting elements of a given group act loxodromically.

Lecture 1: Recall Sageev's construction and focus on the median perspective of a CAT(0) cube complex.
Lecture 2: Discuss the more general construction where, starting with a set S and a collection of walls W, one can build actions on various types of metric spaces.
Lecture 3: Give a detailed proof that mapping class groups admit actions on injective spaces, relying on the construction from Lecture 2. I will also show how to build a universal hyperbolic space on which all strongly contracting elements of a given group act loxodromically.
Lecture 4: Show that the curve complex of a mapping class group admits an equivariant quasi-isometric embedding into a finite product of quasi-trees, again relying on the construction from Lecture 2.

Hotel 1:

FX Hotel ZhongGuanCun (富驿时尚酒店中关村店).

Address: No.68 North 4th Ring West Road (South of Haidian Bridge) Haidian District, Beijing 100080 China.



Hotel 2:

Zhongguan Xinyuan Global Village PKU (北京大学中关新园).

Address: 126 Zhongguancun N Ave, Haidian District, Beijing, Chine, 100871