Lecturer: Mark Hagen (University of Bristol) Title: Real cubings and their applications Abstract:

I will start with a short introduction to median metric spaces (a common generalisation of CAT(0) cube complexes and real trees, among other examples), and also define coarse median spaces as defined by Bowditch. Next, I will recall the class of hierarchically hyperbolic spaces (HHS), which are a subclass of coarse median spaces. There is a fine-geometric analogue: if one imposes some additional conditions on a median space --- analogous to the conditions on a coarse median space needed to make an HHS --- one gets the notion of a real cubing; these will be the main objects in the course.

After defining them, the goal will be explaining two things: (1) a sketch of the proof that any asymptotic cone of an HHS is bilipschitz equivalent to a real cubing, and (2) why the extra structure in a real cubing, beyond just the median structure, is desirable.

The minicourse will use results from many authors; the notion of a real cubing comes from joint work with M. Casals-Ruiz and I. Kazachkov, a draft of which is accessible here: https://www.wescac.net/cones_july_2024-public.pdf.

In terms of that document, the course will cover a subset of Sections 4-7, 12-17, and a sketch of the results in Part 4.

Lecturer: Vadim Kaimanovich (University of Ottawa)

Title: Boundary measures on hyperbolic groups

Abstract:

The presence of a rich topological boundary is an indispensable feature of hyperbolic groups. The purpose of this mini-course is to give an introduction to the approach based on considering various natural measures or measure classes on the hyperbolic boundary.

I will begin with an overview of the Rokhlin theory of standard measure spaces, and will further look at the basic properties and interrelations of boundary measures arising from various dynamical considerations: Hausdorff and conformal measures, geodesic currents, and harmonic measures of random walks.

Lecturer: Anders Karlsson (Université de Genève)

Title: Metric functionals and their applications

Abstract:

This mini-course will develop a part of metric geometry in analogy with functional analysis. Like the linear theory there is the important concept of functionals (Busemann and horofunctions) and spectral theorems (for nonexpansive maps and an ergodic theorem for random products of maps. Isometries moreover enjoy a fixed point theorem). Examples and applications include finitely generated groups, invertible linear operators, holomorphic maps, surface homeomorphisms, certain neural networks, and

reinforcement learning.

References:

1. Gouëzel, S; Karlsson, A. Subadditive and multiplicative ergodic theorems, J. Eur. Math. Soc. 22 (2020), no. 6, 1893–1915.

2. Karlsson, Anders, From linear to metric functional analysis. Proc. Natl. Acad. Sci. USA 118 (2021), no. 28, Paper No. e2107069118, 5 pp.

3. Karlsson, Anders, A metric fixed point theorem and some of its applications. Geom. Funct. Anal.34(2024), no.2, 486–511.

Lecturer: Alessandro Sisto (Heriot-Watt University) Title: Introduction to hierarchical hyperbolicity Abstract:

Hierarchically hyperbolic spaces were introduced about ten years ago as a common framework to understand the geometry of mapping class groups and CAT(0) cube complexes, and since then many other examples, as well as applications, have been found. I will give an introduction to this notion, emphasising connections with cubical geometry, specifically via coarse medians and the cubical approximation theorem (both relevant for the other two mini-courses).

Main source: What is a hierarchically hyperbolic space? <u>https://arxiv.org/abs/1707.00053</u>

Lecturer: Abdul Zalloum (Harbin Institute of Technology) TBA