Courses

Lecturer: Cyril Labbé (Paris Cité)

Title: On the Schrödinger operator with white noise potential

Abstract: The goal of this course will be to present recent progress on our understanding of the operator obtained by perturbing the Laplacian with a white noise, in dimensions 1, 2 and 3. This random Schrödinger operator appears formally as a scaling limit of the discrete Laplacian perturbed by a field of i.i.d. random variables. In this course, we will present several aspects of this operator:

(1) its construction, which in dimension 1 relies on the theory of Sturm-Liouville operators, while it requires in dimensions 2 and 3 to apply renormalisation techniques that were developed in the recent theories of stochastic PDEs (regularity structures and paracontrolled distributions).

(2) its connection with the parabolic Anderson model, which has been studied extensively in the discrete setting.

(3) the study of its spectrum, which lets appear a variety of behaviors and displays connections with recent works in Random Matrix Theory.

The course will require some background in probability theory, but will not assume familiarity with spectral theory of self-adjoint operators.

Lecturer: Nikolay Tzvetkov (ENS Lyon)

Title: Growth of high Sobolev norms, invariant and quasi-invariant measures for the defocusing nonlinear Schrödinger equation

Abstract: In these lectures we consider the cubic defocusing nonlinear Schrödinger (NLS) equation, posed on various spatial domains. Thanks to the Hamiltonian structure, the Sobolev norm H^1 remains bounded for all times under the dynamics of the cubic defocusing NLS. It is however not clear what happens with the high Sobolev norms H^s for $s > 1$.

We will first show that in the case when the cubic defocusing NLS is posed on the three dimensional euclidean space $\mathbf{R}^3$, then the high Sobolev norms remain bounded for all

times. We will next show that the high Sobolev norms may not remain bounded when the cubic defocusing NLS is posed on the three dimensional manifold $\mathbf{R} \times \mathbf{T}^2$. We will also discuss upper bounds on the high Sobolev norms and many open problems concerning the growth of high Sobolev norms for the defocusing NLS.

In the second part of the lectures we will consider the evolution of Gaussian fields under the cubic defocusing NLS, posed on a compact Riemannian manifold. We will first consider the flat torus as a spatial domain and we will show that the Sobolev norms of the solutions enjoy very good bounds in the case when the data is distributed according to the Gibbs measure. We will then show that the same problem posed on the two dimensional standard sphere offers remarkable new phenomena.