Winter School on Complex Dynamics and Related Fields

19–23 January, 2025

Lecture Hall, Jiayibing Building, Jingchunyuan 82, BICMR, Peking University

Organizing Committee

Hanlong Fang (Peking University, China)

Zhiqiang Li (Peking University, China)

Lecturers

Walter Bergweiler (Christian-Albrechts-Universität zu Kiel, Germany)

Yunping Jiang (City University of New York, USA)

Sergiy Merenkov (City University of New York, USA)

Daniel Meyer (University of Liverpool, UK)

	Sunday 19-1月-25	Monday 20-1月-25	Tuesday 21-1月-25	Wednesday 22-1月-25	Thursday 23-1月-25
8:30 - 9:00	Registration				
	Speaker	Speaker	Speaker	Speaker	Speaker
9:00 - 10:00	Yunping Jiang	Sergiy Merenkov	Daniel Meyer	Daniel Meyer	Walter Bergweiler
10:00 - 10:30	Tea Break				
10:30 - 11:30	Walter Bergweiler	Yunping Jiang	Yunping Jiang	Yunping Jiang	Daniel Meyer
11:30 - 14:00	Lunch				
14:00 - 15:00	Daniel Meyer	Walter Bergweiler	Free Discussions	Walter Bergweiler	Free Discussions
15:00 - 15:30	Break			Break	
15:30 - 16:30	Sergiy Merenkov	Daniel Meyer		Sergiy Merenkov	
16:30 onwards	Free Discussions	Free Discussions		Free Discussions	

ABSTRACT

Walter Bergweiler

Affiliation: Christian-Albrechts-Universität zu Kiel, Germany

Title: The Escaping Set of an Entire Function

Abstract:

The escaping set of an entire function consists of all points in the complex plane which tend to infinity under iteration. This set plays a central role in the dynamics of transcendental entire functions. The goal of this lecture series is to explain this role and to discuss some of the main results in the area. In particular, we will discuss the connection between the Julia set and the escaping set. This includes a discussion of the so-called Eremenko-Lyubich class, for which the escaping set is a subset of the Julia set. We will also study the rates at which points in the escaping set tend to infinity, and here in particular the so-called fast escaping set. Further topics are topological properties of the escaping set and the Hausdorff dimension and Lebesgue measure of the escaping set. We will also discuss escaping Fatou components. This includes multiply connected Fatou components.

Yunping Jiang

Affiliation: City University of New York, USA

Title: Holomorphic Families of Injections Over Riemann Surfaces and Monodromy

Abstract:

This lecture series explores holomorphic families of injective mappings, or holomorphic motions, over hyperbolic Riemann surfaces, using the framework of generalized Beltrami equations. The series consists of four lectures:

Lecture 1: Quasiconformal Mappings and the Measurable Riemann Mapping Theorem.

Abstract: This lecture introduces quasiconformal mappings through Grötzsch’s argument and examines the measurable Riemann mapping theorem (MRMT). It emphasizes the singular integral techniques to the proof of MRMT, which serve as a foundation for the discussions in Lectures 3 and 4.

Lecture 2: Holomorphic Motions and the λ-Lemma.

Abstract: This lecture investigates holomorphic motions of subsets of the Riemann sphere over hyperbolic Riemann surfaces. The λ-Lemma is stated and proved, followed by an exploration of its applications in complex dynamics.

Lecture 3: Holomorphic Motions over the Unit Disk and Chirka's Method.

Abstract: This lecture delves into holomorphic motions of subsets of the Riemann sphere over the unit disk, incorporating insights from Bers and Royden. Using Chirka's method, which relies on singular integrals discussed in Lecture 1, Slodkowski's theorem is proved.

Lecture 4: Winding Number, Monodromy, and Guiding Isotopy

Abstract: This lecture explores Bers and Royden's framework, which treats a holomorphic motion of a subset of the Riemann sphere over a hyperbolic Riemann surface as the converse to the measurable Riemann mapping theorem (MRMT). A counterexample originally constructed by Douady and later modified by Earle is introduced and confirmed as a valid counterexample through its non-zero winding numbers. A distinct counterexample with non-zero winding numbers, which we have constructed, will also be presented. The lecture then introduces a new counterexample, which I have constructed, featuring a holomorphic motion with zero winding numbers. To establish this as a counterexample, the concept of monodromy is introduced and it is demonstrated that this example exhibits non-trivial monodromy, confirming it as a counterexample. Finally, the lecture concludes with a discussion of our result, showing that the trivial monodromy condition (or equivalently, the guiding isotopy condition) is both necessary and sufficient for a holomorphic motion of a subset of the Riemann sphere over a hyperbolic Riemann surface to align with Bers and Royden's perspective as the converse to MRMT.

Sergiy Merenkov

Affiliation: City University of New York, USA

Title: Wandering Domains in Smooth Dynamics

Abstract:

1. Denjoy diffeomorphisms of tori and round wandering domains.

2. Surface diffeomorphisms permuting a dense collection of domains with bounded geometry.

3. A question of Kwakkel and Markovic on C^1-regularity.

References:

1. F. Kwakkel, V. Markovic, Topological entropy and diffeomorphisms of surfaces with wandering domains, Ann. Acad. Sci. Fenn. Math. 35 (2010), no. 2, 503–513.

2. P. D. McSwiggen, Diffeomorphisms of the torus with wandering domains, Proc. Amer. Math. Soc. 117 (1993), no. 4, 1175–1186.

3. S. Merenkov, No round wandering domains for C 1 -diffeomorphisms of tori, Ergodic Theory Dynam. Systems 39 (2019), no. 11, 3127–3135.

4. S. Merenkov, On a question of Kwakkel and Markovic. https://arxiv.org/abs/2405.02176.

5. A. Navas, Wandering domains for diffeomorphisms of the k-torus: a remark on a theorem by Norton and Sullivan, https://arxiv.org/abs/1702.02251.

6. A. Norton, D. Sullivan, Wandering domains and invariant conformal structures for mappings of the 2-torus, Ann. Acad. Sci. Fenn. Math. 21 (1996), no. 1, 51–68.

Daniel Meyer

Affiliation: University of Liverpool, UK

Title: Mating and Unmating in Complex Dynamics

Abstract:

Mating is an operation that combines the Julia sets of two polynomials in a geometric fashion. This results in a new dynamical system. Often, but not always, this new dynamical system is (conjugate to) a rational (i.e. holomorphic) map. Unmating is the reverse operation, namely given a rational map, one wishes to find polynomials that mate to the given map.

This mini-course gives an introduction to this subject. A broad outline of the material is as follows.

1. Background from Complex Dynamics, in particular Boettcher's theorem and external rays.

2. The different definitions of mating, in particular the formal, the topological, and the essential mating.

3. Obstructions to matings, things that prevent the mating from being rational, in particular

Moore's theorem and Thurston's characterization of rational maps.

4. The Rees-Tan Lei-Shishikura theorem on matability in the quadratic case.

5. Unmating, including the algorithm to find the involved polynomials.

Time permitting, we will mention related operations such as the Cannon-Thurston Peano curves (in the setting of Kleinian groups) as well as mating/unmating of random trees (in the setting of the Brownian map) and variants of mating (such as Wolf Jungs anti-mating).