北京国际数学研究中心
BEIJING INTERNATIONAL CENTER FOR MATHEMATICAL RESEARCH

Summer School on Propagation of Chaos in Analysis and Probability

Time: August 5-9, 2024

Venue: Lecture Hall, Jiayibing Building, Jingchuyuan 82

Beijing International Center for Mathematical Research

Peking University

List of Talks

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<tr><td></td><td>Aug.5 (Mon)</td><td>Aug.6 (Tue)</td><td>Aug.7 (Wed)</td><td>Aug.8 (Thu)</td><td>Aug.9 (Fri)</td></tr>
<tr><td>8:50am-
9:00am</td><td>Registration
Reception</td><td colspan="4"></td></tr>
<tr><td>Morning
Chair</td><td>Zhenfu Wang</td><td>Xicheng Zhang</td><td>Wei Liu(JSNU)</td><td>Zhenjie Ren</td><td>Jian Song</td></tr>
<tr><td>9:00am-
10:50am</td><td>Lecture 1-2
Feng-Yu Wang</td><td>Lecture 1-2
Liming Wu</td><td>Lecture 3-4
Xicheng Zhang</td><td>Lecture 1-2
Peter Pickl</td><td>Lecture 3-4
Peter Pickl</td></tr>
<tr><td>10:50am-
11:10am</td><td colspan="5">Tea Break</td></tr>
<tr><td>11:00am-
noon</td><td>Invited talk
Xing Huang</td><td>Invited talk
Wei Liu(JSNU)</td><td>Invited talk
Zhenjie Ren</td><td>Invited talk
Wei Liu(WHU)</td><td>Invited talk
Ning Jiang</td></tr>
<tr><td>noon-
2:00pm</td><td colspan="5">Lunch Break</td></tr>
<tr><td>Afternoon
Chair</td><td>Feng-Yu Wang</td><td>Liming Wu</td><td rowspan="4">Free afternoon</td><td>Wei Liu(WHU)</td><td>Zhenfu Wang</td></tr>
<tr><td>2:00pm-
3:50pm</td><td>Lecture 1-2
Xicheng Zhang</td><td>Lecture 3-4
Feng-Yu Wang</td><td>Lecture 3-4
Liming Wu</td><td rowspan="3">Invited talks
2:00pm-2:40pm
Zimo Hao
2:40pm-3:10pm
Tea break
3:10pm-3:50pm
Wangjun Yuan
3:50pm-4:30pm
Xianliang Zhao</td></tr>
<tr><td>3:50pm -
4:10pm</td><td colspan="2">Tea Break</td><td>Tea break</td></tr>
<tr><td>4:10pm-
5:00pm</td><td>Invited talk
Lei Li</td><td>Invited talk
Shuyang Ling</td><td>Invited talk
Jian Song</td></tr>
<tr><td>6:00pm -
8:00pm</td><td>Free evening</td><td>Banquet</td><td colspan="3">Free evening</td></tr>
</table>

- **Summer school location:** Lecture Hall, Jiayibing Building, Jingchunyuan 82, BICMR, Peking University.
- **Hotel location:** 北京大学中关新园(Zhongguanyuan Global Village): No.126, Zhongguancun North Street, Haidian, Beijing
- **Banquet location:** 北京大学勺园中餐厅(Shaoyuan Restaurant): No.5 Yiheyuan Road, Haidian, Beijing.
- **Contact:** Ms. He Liu, Tel: +86 10 62744134 or Email: liuhe@math.pku.edu.cn

Abstracts of Talks

Quantitative Propagation of Chaos in Wasserstein Distance for Mean Field Particle System

Xing Huang
Tianjin University

In this talk, quantitative propagation of chaos in L^η-Wasserstein distance with $\eta \in (0,1)$ for mean field interacting particle system is derived, where the diffusion coefficient is allowed to be interacting and the initial distribution of interacting particle system converges to that of the limit equation in L^1-Wasserstein distance. The non-degenerate and degenerate cases are investigated respectively and the main tool relies on the gradient estimate of the decoupled SDEs.

Supercritical McKean-Vlasov SDE driven by cylindrical alpha-stable process: well-posedness, propagation of chaos and Euler's approximation

Zimo Hao
Bielefeld University

We study the well-posedness, propagation of chaos and the Euler approximation of the following supercritical McKean-Vlasov stochastic differential equation driven by a symmetric non-degenerate cyclindrical α-stable process in $\mathbb{R}^d$ with $\alpha \in (0,1)$:

$$\mathrm{d}X_t = (K * \mu_t)(X_t)\mathrm{d}t + \mathrm{d}L_t^{(\alpha)}, \quad X_0 = x \in \mathbb{R}^d,$$

where $K : \mathbb{R}^d \to \mathbb{R}^d$ belongs to the usual β-order Hölder space with some $\beta > 1-\alpha$, and μ_t is the time marginal distribution of the solution. The weak and strong well-posedness are established under $\beta \in (1-\alpha, 1)$ and $\beta \in (1-\alpha/2, 1)$ respectively by Picard's iteration and the Hölder regularity of associated supercritical partial differential equations, and then the weak and strong propagation of chaos and the Euler approximation are obtained via the classical martingale approach and the Itô-Tanaka trick. This is a joint work with Chongyang Ren and Mingyan Wu.

The mean field limit of random batch interacting particle systems

Lei Li
Shanghai Jiaotong University

The Random Batch Method proposed in our previous work (Jin et al. J Comput. Phys, 2020) is not only a numerical method for interacting particle systems and its mean-field limit, but also can be viewed as a new model in which particles interact, at discrete time, with randomly selected mini-batch of particles. We investigate the mean-field limit of this model as the number of particles tends to infinity. The mean field limit now exhibits some new features. We will not only justify this mean-field limit (discrete in time) but will also show that the limit approaches to the solution of a nonlinear Fokker-Planck equation as the discrete time step goes to zero.

Local geometry determines global landscape in low-rank factorization for synchronization

Shuyang Ling
NYU Shanghai

The orthogonal group synchronization problem, which focuses on recovering orthogonal group elements from their corrupted pairwise measurements, encompasses examples such as high-dimensional Kuramoto model on general signed networks, $\mathbb{Z}_2$-synchronization, community detection under stochastic block models, and orthogonal Procrustes problem. The semidefinite relaxation (SDR) has proven its power in solving this problem; however, its expensive computational costs impede its widespread practical applications. We consider the Burer-Monteiro factorization approach to the orthogonal group synchronization, an effective and scalable low-rank factorization to solve large scale SDPs. Despite the significant empirical successes of this factorization approach, it is still a challenging task to understand when the nonconvex optimization landscape is benign, i.e., the optimization landscape possesses only one local minimizer, which is also global. In this work, we demonstrate that if the degree of freedom within the factorization exceeds twice the condition number of the "Laplacian" (certificate matrix) at the global minimizer, the optimization landscape is absent of spurious local minima. Our main theorem is purely algebraic and versatile, and it seamlessly applies to all the aforementioned examples: the nonconvex landscape remains benign under almost identical condition that enables the success of the SDR.

Well-posedness and Propagation of Chaos of Mckean-Vlasov SPDEs

Wei Liu
Jiangsu Normal University

In this talk we mainly present some well-posedness and propagation of chaos results for a class of McKean-Vlasov SPDEs. As application we provide some illustrative examples such as the weakly coupled Allen-Cahn equations, the weakly coupled Burgers type equations and the weakly coupled 2D Navier-Stokes equations.

Long time behaviors and Kramers' type law for two-species nonlinear McKean-Vlasov system

Wei Liu
Wuhan University

In this talk, we will show the long time behaviors and exit-time problem of the two-species nonlinear McKean-Vlasov system. Under two new assumptions regarding the magnitude of the second-order mixed partial derivative $\nabla^2_{xy}W$ of interaction potential W, we obtain the exponential convergence, concentration inequalities, uniform in time propagation of chaos by using the coupling method, which extend the results of Liu-Wu-Zhang (CMP 21') from one-species system to two-species system. We also study the creation of chaos, the weak version of Kramers' type law and the exit-time problem.

Fluid limits from Boltzmann equations

Ning Jiang
Wuhan University

The Boltzmann equation is a fundamental equation in non-equilibrium statistical physics, which describes the dynamics of the dilute gases. Under different physical scalings, it can be derived several macroscopic fluids equations from the Boltzmann equation: compressible (incompressible) Navier-Stokes and Euler equations, and their corresponding boundary conditions. The mathematically rigorous justifications of these process are closely related to the famous Hilbert 6th problem. In this talk, we introduce the formal derivations of these fluid equations, and review some most important mathematical results in this field in the past 40 years.

Derivation of effective equations for interacting many particle system

Peter Pickl
Universität Tübingen, Germany

The numerical and analytical treatment of systems of many interacting particles is often very difficult, if not impossible. However, in some situation one can find an effective description of the systems with much less degrees of freedom. Depending on the microscopic model considered, one arrives at the Vlasov equation, Fokker-Planck equation or others.
During the lectures we will discuss, first on a heursitic level, how one finds the effective description for an interacting many body system. I will focus on a system of weakly interacting, Newtonian particles which is effectively well described by the so-called "Vlasov-equation". Different types of convergence of the microscopic system to the effective description will be formulated. Based on this different techniques on how to prove the validity of the effective description will be presented. Among other we will see, how an additional white-noise terms helps in proving the validity of the effective description.

Self-interacting approximation to McKean-Vlasov long time limit

Zhenjie Ren
University Paris-Dauphine, PSL

We investigate the smoothing estimates for the non-cutoff Boltzmann equation with soft potentials as well as Landau-Coulomb equation in L^2 framework. We address the problem in two different settings: (i). When the initial data only possesses finite polynomial moment, the solutions to the Boltzmann equation have only finite Sobolev regularity while the solutions to the Landau-Coulomb have the infinite Sobolev regularity but with negative weight. (ii). When the initial data have exponential moments,the solutions belong to the Gevrey class with an optimal index that depends on the exponential moment for any positive time.

High dimensional limits for a class of particle systems

Jian Song
Shandong University

For a class of particle systems that generalizes the eigenvalues of a class of matrix-valued processes, we study the high dimensional limits of the empirical measures. The talk is based on joint works with Jianfeng Yao and Wangjun Yuan.

Regularity and Ergodicity for Distribution Dependent SDEs

Feng-Yu Wang
Tianjin University

This mini course introduces some recent progresses we made on the regularity and ergodicity of distribution dependent stochastic differential equations. Main contents include:

1. The log-Harnack inequality (inverse Talagrand inequality);
2. Bismut formula for intrinsic/Lions derivatives;
3. Ergodicity in Wasserstein distance and entropy.

Functional and concentration inequalities for interacting particles systems and applications to McKean-Vlasov equation

Liming Wu
Université Clermont-Auvergne & Harbin Institute of Technology

In this series of talks, I will present some basic ideas and appoaches to study the behaviors of Gibbs measures and the associated Glauber dynamics (systems of interacting particles). The basic ideas are Wasserstein distance, Dobrushin's interdependence coefficients and uniqueness condition and relative entropy. The basic approaches are Poincare inequality, logarithmic Sobolev inequality, transport-entropy inequality (Talagrand) or transport-information inequality. From those functional inequalities we can derive concentration inequality for systems of interacting particles, which yield quantitative estimates of propagation of chaos in the mean field case. Those functional inequalities have also immediate consequences for the exponential convergence of the McKean-Vlasov equation to its equilibrium.

Eigenvalue distributions of high-dimensional matrix processes driven by fractional Brownian motion

Wangjun Yuan
University of Luxembourg

We study high-dimensional behavior of empirical spectral distributions $\{L_N(t), t \in [0,T]\}$ for both Wigner-type and Wishart-type matrices, whose entries are generated from the solution of stochastic differential equation driven by fractional Brownian motion with Hurst parameter $H \in (1/2, 1)$. The limits of $\{L_N(t), t \in [0,T]\}$ for both Wigner-type and Wishart-type matrices as $N \to \infty$ are also characterised. This is a joint work with Jian Song and Jianfeng Yao.

Propagation of chaos for moderately interacting particle systems related to singular kinetic McKean-Vlasov SDEs

Xicheng Zhang
Beijing Institute of Technology

In this short course, I will introduce second-order singular McKean-Vlasov stochastic differential equations (SDEs) and discuss the associated propagation of chaos. We establish the local and global well-posedness of weak and strong solutions for second-order fractional mean-field SDEs. These equations involve singular or distribution interaction kernels and measure initial values, with examples including Newton or Coulomb potentials, Riesz potentials, Biot-Savart law, among others. Our analysis relies on the theory of anisotropic Besov spaces. Building on the well-posedness results of the McKean-Vlasov equations, we investigate the propagation of chaos for moderately interacting particle systems with singular kernels and derive quantitative convergence rates. Time permitting, I will also introduce a novel algorithm for diffusion models that is closely related to the particle system.

Graphon Particle Systems with Singular Kernels

Xianliang Zhao
Bielefeld University & AMSS of Chinese Academy of Sciences

In the talk, we shall study stochastic graphon particle systems with L^p-kernels. We will show the non-exchangeable mean-field limits and graph limits, deriving associated nonlinear Fokker-Planck equations and Euler-type equations. The method is based on the tightness argument in [arXiv:2209.14002, 2022] for non-exchangeable interacting diffusions, which involves analyzing the systems's Fisher information and propagating the regularities.

List of Speakers

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