Titles and Abstracts

Lecturer: Claudio Landim

Title: Metastable Markov chains

Abstract:

We review recent results on the metastable behavior of continuous-time Markov chains derived through the characterization of Markov chains as unique solutions of martingale problems.

We first provide a necessary and sufficient condition for the metastability of a Markov chain, expressed in terms of a property of the solutions of the resolvent equation.

As an application, we consider the elliptic operator given by

$$L_\epsilon f \;=\; \boldsymbol{b}\cdot\nabla f \;+\; \epsilon\,\Delta f$$

for some smooth vector field $\boldsymbol{b}\colon R^d \to R^d$ and a small parameter $\epsilon > 0$. Consider the initial-valued problem

$$\begin{cases} \partial_t u_\epsilon \;=\; L_\epsilon u_\epsilon \;, \\ u_\epsilon(0,\cdot) = u_0(\cdot) \;, \end{cases}$$

for some bounded continuous function u_0. Under the hypothesis that $\boldsymbol{b} = -(\nabla U + \ell)$, where ℓ is a divergence-free field orthogonal to ∇U, we examine the asymptotic behavior of the solution u_ϵ in different time-scales.

Lecturer: Ron Peled

Title: Disordered spin systems, First-passage percolation and minimal surfaces in random environment

Abstract:

The course will explore various ways in which disorder (i.e., a random environment) alters the behavior of familiar models. Emphasis will be put on the many open questions in the field, which the audience is encouraged to contemplate. The three talks are independent and attendance of one is not a prerequisite for attending others.

Lecture 1 (disordered spin systems): What is the effect of adding a small random field to a statistical physics model? Imry-Ma (1975) predicted that such a random field causes the Ising model to lose its magnetized phase in two dimensions, and that it causes the XY model to lose its magnetized phase in all dimensions up to four. This was proved by Aizenman-Wehr (1989). We discuss the Imry-Ma phenomenon and its generalizations to other spin systems and explain new quantitative bounds obtained in recent years, with focus on a joint work with Dario and Harel.

Lecture 2 (First-passage percolation): Give each edge of $\mathbf{Z}^d$ a random length, independently sampled from a common distribution (e.g., uniform[1,2]). This turns $\mathbf{Z}^d$ into a random metric space. First-passage percolation studies the geometry and length of long geodesics in this metric. We describe the (still poor) state-of-the art and give ideas from a recent proof for the coalescence of geodesics in two dimensions, joint with Dembin and Elboim.

Lecture 3 (minimal surfaces in random environment): Assign each edge of $\mathbf{Z}^d$ a random capacity, independently sampled from a common distribution. How do minimal surfaces in this random environment look like? (when $d = 2$, minimal surfaces become geodesics in first-passage percolation so we focus on $d \geq 3$) We present the physics predictions, and give ideas from a recent proof of localization of certain minimal surfaces in dimensions d¿=4, joint with Bassan and Gilboa. The latter result is applied to prove the existence of non-constant ground configurations in the disordered Ising ferromagnet.