



# Recent advances in kinetic theory and related models

**Time: December 4 - December 8, 2023**

**Venue: Lecture Hall, Jiayibing Building, Jingchunyuan 82**

**Beijing International Center for Mathematical Research  
Peking University**



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	Morning Chair: Zhennan Zhou  Afternoon Chair: Ruo Li	Morning Chair: Zhenfu Wang	Morning Chair: Yu Feng  Afternoon Chair: Zhifei Zhang	Morning Chair: Shi Jin
8:30am- 9:30am		Lingbing He	Enrique Zuazua	Francis Filbet
	Opening (9:15am-9:30am)			
9:30am- 10:30am	Jose Carrillo	Renjun Duan	Giovanni Russo	Yanli Wang
10:30am- 11:00am	Tea Break			
11:00am- noon	Seung-Yeal Ha	Shiwu Yang	Lorenzo Pareschi	Liu Liu
noon- 2:00pm	Lunch Break			Closing (noon-12:15pm)
				Lunch Break
2:00pm- 3:00pm	Xicheng Zhang	Free afternoon	Christopher Henderson	Free afternoon
3:00pm- 4:00pm	Ning Jiang		Young-Pil Choi	
4:00pm- 4:30pm	Tea Break		Tea Break	
4:30pm- 5:30pm	Xuecheng Wang		Haitao Wang	

# ABSTRACTS OF TALKS

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## NOISE-DRIVEN BIFURCATIONS IN A NEURAL FIELD SYSTEM MODELLING NETWORKS OF GRID CELLS

**Jose Carrillo**  
University of Oxford

In this talk I will review several results in the modelling of grid cells. The activity generated by an ensemble of neurons is affected by various noise sources. It is a well-recognised challenge to understand the effects of noise on the stability of such networks. We demonstrate that the patterns of activity generated by networks of grid cells emerge from the instability of homogeneous activity for small levels of noise. This is carried out by upscaling a noisy grid cell model to a system of partial differential equations in order to analyse the robustness of network activity patterns with respect to noise. This is rigorously achieved by mean-field type arguments. Inhomogeneous network patterns are numerically understood as branches bifurcating from unstable homogeneous states for small noise levels. We prove that there is a phase transition occurring as the level of noise decreases. Our numerical study also indicates the presence of hysteresis phenomena close to the precise critical noise value. This talk is a summary of four papers/preprints in collaboration with A. Clini, H. Holden, P. Roux and S. Solem.

## QUANTIFIED OVERDAMPED LIMIT FOR VLASOV-FOKKER-PLANCK EQUATIONS WITH SINGULAR INTERACTION FORCES

**Young-Pil Choi**  
Yonsei University

In this talk, I will discuss a quantified overdamped limit for kinetic Vlasov-Fokker-Planck equations with nonlocal interaction forces. We provide explicit bounds on the error between solutions of that kinetic equation and the limiting equation, which is known under the names of aggregation-diffusion equation or McKean-Vlasov equation. Our strategy only requires weak integrability of the interaction potentials, thus in particular it includes the quantified overdamped limit of the kinetic Vlasov-Poisson-Fokker-Planck system to the aggregation-diffusion equation with either repulsive electrostatic or attractive gravitational interactions.

## TWO DOUBLE-PARAMETERS LIMIT PROBLEMS ON THE VLASOV-POISSON-LANDAU SYSTEM

**Renjun Duan**  
The Chinese University of Hong Kong

The Vlasov-Poisson-Landau system and the Euler-Poisson system are both fundamental models in plasma physics describing the complex dynamics of plasmas under the selfconsistent electric interactions at the kinetic and fluid levels, respectively. It is well known that the formal Hilbert expansion of VPL system gives the Euler-Poisson system from which one can further derive the quasi-neutral Euler system by letting the Debye length tend to zero or the KdV equations under the so-called Gardner-Morikawa transformation. In this talk I present recent results on such two double-parameters limit problems directly from the VPL system in the corresponding appropriate scalings. We treat the physically most important Coulomb interaction potentials. The approach of the proof we have adopted is to construct the energy functional and energy dissipation functional via the macro-micro decomposition capturing the singularity of those fluid limit problems with respect to two small parameters dissipation strength vs dispersion strength. Joint with Dongcheng Yang and Hongjun Yu.

## A STRUCTURE AND ASYMPTOTIC PRESERVING SCHEME FOR THE VLASOV-POISSON-FOKKER-PLANCK MODEL

**Francis Filbet**

Université Toulouse-III-Paul-Sabatier

We propose a numerical method for the Vlasov-Poisson-Fokker-Planck model written as an hyperbolic system thanks to a spectral decomposition in the basis of Hermite functions with respect to the velocity variable and a structure preserving finite volume scheme for the space variable. On the one hand, we show that this scheme naturally preserves both stationary solutions and linearized free-energy estimate. On the other hand, we adapt previous arguments based on hypocoercivity methods to get quantitative estimates ensuring the exponential relaxation to equilibrium of the discrete solution for the linearized Vlasov-Poisson-Fokker-Planck system, uniformly with respect to both scaling and discretization parameters. Finally, we perform substantial numerical simulations for the nonlinear system to illustrate the efficiency of this approach for a large variety of collisional regimes (plasma echos for weakly collisional regimes and trend to equilibrium for collisional plasmas) and to highlight its robustness (unconditional stability, asymptotic preserving properties).

## EMERGENT DYNAMICS OF INFINITELY MANY KURAMOTO OSCILLATORS

**Seung-Yeal Ha**

Seoul National University

In this talk, we propose an infinite Kuramoto model for a countably infinite set of Kuramoto oscillators and study its emergent dynamics for two classes of network topologies. For a class of symmetric and row (or column)-summable network topology, we show that a homogeneous ensemble exhibits complete synchronization, and the infinite Kuramoto model can cast as a gradient flow, whereas we obtain a weak synchronization estimate, namely practical synchronization for a heterogeneous ensemble. Unlike with the finite Kuramoto model, phase diameter can be constant for some class of network topologies which is a novel feature of the infinite model. We also consider a second class of network topology (so-called a sender network) in which coupling strengths are proportional to a constant that depends only on sender's index number. For this network topology, we have a better control on emergent dynamics. For a homogeneous ensemble, there are only two possible asymptotic states, complete phase synchrony or bi-cluster configuration in any positive coupling strengths. In contrast, for a heterogeneous ensemble, complete synchronization occurs exponentially fast for a class of initial configuration confined in a quarter arc. This is a joint work with Euntaek Lee (SNU) and Woojoo Shim (Kyungpook National University).

## SHARP REGULARITY ESTIMATES FOR THE COLLISIONAL KINETIC EQUATIONS

**Lingbing He**

Tsinghua University

We investigate the smoothing estimates for the non-cutoff Boltzmann equation with soft potentials as well as Landau-Coulomb equation in  $L^2$  framework. We address the problem in two different settings: (i). When the initial data only possesses finite polynomial moment, the solutions to the Boltzmann equation have only finite Sobolev regularity while the solutions to the Landau-Coulomb have the infinite Sobolev regularity but with negative weight. (ii). When the initial data have exponential moments, the solutions belong to the Gevrey class with an optimal index that depends on the exponential moment for any positive time.

## THE BOLTZMANN EQUATION WITH LARGE DATA

**Christopher Henderson**

University of Arizona

The Boltzmann equation is a nonlocal, nonlinear equation arising in gas dynamics for which (large data) global well-posedness is an extremely difficult problem that is nearly completely open. In this talk I will discuss a recent program to understand a more tractable, related question: what is the largest space in which local well-posedness holds and what quantities prevent blow-up at finite times? This program covers several papers and intertwines with a recent push to understand the regularity theory of kinetic Fokker-Planck-type equations. I will give a broad outline of the proof, and then, instead of slogging through the whole thing, I will focus on a simple proof of a technically important and physically interesting lower bound – how “vacuum regions” fill in. This is a joint work with Snelson and Tarfulea.

## COMPRESSIBLE EULER LIMIT FROM BOLTZMANN EQUATION IN DOMAINS WITH BOUNDARY

**Ning Jiang**

Wuhan University

It is a central problem in kinetic theory that the limit from the Boltzmann equations in domains with boundary to compressible Euler limit (or its linearized version, acoustic system). The key issue is to derive the boundary conditions of the fluid system from those of the Boltzmann equations, for example, Maxwell reflection or incoming boundary conditions. Since the limiting Maxwellian usually does not match the boundary condition of the original Boltzmann equations, Knudsen and Prandtl boundary layers are generated. We will report some recent progress on this problem.

## MULTI-FIDELITY METHOD FOR A CLASS OF KINETIC PROBLEMS WITH UNCERTAINTIES

**Liu Liu**

The Chinese University of Hong Kong

In this talk, we will discuss some work on multi-fidelity methods for solving a class of kinetic models with uncertainties and multiple scales. The Boltzmann equation, linear transport equation, Vlasov-Poisson system, semi-classical Schrödinger equation and epidemic transport system will be particularly studied. We will provide some formal error analysis. Numerical experiments will be presented to justify the robustness and accuracy of the numerical schemes. These are joint works with Lorenzo Pareschi, Xueyu Zhu, Giulia Bertaglia and Yiwen Lin.

## GRADIENT-BASED MONTE CARLO METHODS FOR HYPERBOLIC AND KINETIC EQUATIONS

**Lorenzo Pareschi**

HWU Edinburgh & University of Ferrara

## HIGH ORDER CONSERVATIVE SEMI-LAGRANGIAN SCHEMES FOR KINETIC EQUATIONS

**Giovanni Russo**

Università di Catania

## STABILITY OF BACKGROUND PERTURBATION FOR BOLTZMANN EQUATION

**Haitao Wang**

Shanghai Jiao Tong University

We will consider the Boltzmann equation in the perturbation regime. Since the macroscopic quantities in the global Maxwellian are obtained through measurements, there are typically some errors involved. We aim to study how the solution changes as the background varies for the same initial perturbation. In this talk, I will show that the solution changes continuously with variations in the global Maxwellian and provide a sharp estimate for the associated errors. The proof relies on refined estimates for the linearized solution operator.

## NONLINEAR STABILITY OF THE VLASOV-POISSON SYSTEM IN $\mathbb{R}^3$

**Xuecheng Wang**

Tsinghua University

We consider the stability problem for the 3D Vlasov-Poisson system in the whole space around the spatially homogeneous nontrivial equilibrium. In particular, we give linear stability for a class of general equilibrium and nonlinear stability for a special equilibrium, which is the so-called Poisson equilibrium. This talk is based on joint works with A. Ionescu (Princeton University), B. Pausader (Brown University), and K. Widmayer (University of Zurich and University of Vienna).

## DATA-DRIVEN NON-LOCAL REDUCED ORDER MODELING FOR RADIATIVE TRANSFER EQUATIONS

**Yanli Wang**

Beijing Computational Science Research Center

In this work, a non-local reduced-order model is built for the radiative transfer equations using the neural network, where the surrogate macroscopic governing equations are solved instead of the original kinetic equations. The Fourier neural operator is utilized when designing the neural network, in which case the global information of all the data is utilized in each local physical position. The boundary condition is also learned using the neural network from the training data. Several benchmark examples such as the Marshak wave and lattice tests validate this new data-driven non-local reduced order modeling.

## ON THE 3D RELATIVISTIC VLASOV-MAXWELL SYSTEM WITH LARGE MAXWELL FIELD

**Shiwu Yang**

Peking University

In this talk, we study the relativistic Vlasov-Maxwell system in space dimension three. We show the global existence of classical solutions for a class of large initial data. The initial Maxwell field is allowed to be arbitrarily large and the initial density distribution is assumed to be small and decay with rate  $(1 + |x| + |v|)^{-9-}$ . This is jointed work with Dongyi Wei.

PROPAGATION OF CHAOS FOR MODERATELY INTERACTING PARTICLE SYSTEMS OF  
SINGULAR KINETIC MCKEAN-VLASOV SDES

**Xicheng Zhang**

Beijing Institute of Technology

In this talk I will report recent works about singular kinetic McKean-Vlasov SDEs. Here the singularity allows us to treat some physical models such as Vlasov-Poisson-Fokker-Planck equations and 2D Navier-Stokes equations, etc. Our computations depends some analytic tools.

CONTROL AND MACHINE LEARNING

**Enrique Zuazua**

Friedrich-Alexander-Universität at Erlangen

In this lecture we shall present some recent results on the interplay between control and Machine Learning, and more precisely, Supervised Learning, Universal Approximation and Normalizing flows. We adopt the perspective of the simultaneous or ensemble control of systems of Residual Neural Networks (ResNets). Roughly, each item to be classified corresponds to a different initial datum for the Cauchy problem of the ResNets, leading to an ensemble of solutions to be driven to the corresponding targets, associated to the labels, by means of the same control. We present a genuinely nonlinear and constructive method, allowing to show that such an ambitious goal can be achieved, estimating the complexity of the control strategies. We shall also present the transport equations counterparts and discuss the links with the theory of optimal transport.



# LIST OF SPEAKERS

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Dec. 4th (Mon)			
Time	Speaker	Title	Host
9:00am-9:30am	Opening		
9:30am-10:30am	Jose Carrillo	Noise-driven bifurcations in a neural field system modelling networks of grid cells	Zhennan Zhou
10:30am-11:00am	Tea Break		
11:00am-noon	Seung-Yeal Ha	Emergent dynamics of infinitely many Kuramoto oscillators	
noon-2:00pm	Lunch Break		
2:00pm-3:00pm	Xicheng Zhang	Propagation of chaos for moderately interacting particle systems of singular kinetic McKean-Vlasov SDEs	Ruo Li
3:00pm-4:00pm	Ning Jiang	Compressible Euler limit from Boltzmann equation in domains with boundary	
4:00pm-4:30pm	Tea break		
4:30pm-5:30pm	Xuecheng Wang	Nonlinear stability of the Vlasov-Poisson system in $R^3$	
6:00pm-8:00pm	Banquet		

Dec. 5th (Tue)			
Time	Speaker	Title	Host
8:30am-9:30am	Lingbing He	Sharp regularity estimates for the collisional kinetic equations	Zhenfu Wang
9:30am-10:00am	Tea Break		
10:00am-11:00am	Renjun Duan	Two double-parameters limit problems on the Vlasov-Poisson-Landau system	
11:00am-noon	Shiwu Yang	On the 3D Relativistic Vlasov-Maxwell System with large Maxwell field	
noon-2:00pm	Lunch Break		
2:00pm-6:00pm	Free Afternoon		





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Dec. 6th (Wed)			
Time	Speaker	Title	Host
8:30am-9:30am	Enrique Zuazua	Control and Machine Learning	Yu Feng
9:30am-10:00am	Tea Break		
10:00am-11:00am	Giovanni Russo	High order conservative Semi-Lagrangian schemes for kinetic equations	
11:00am-noon	Lorenzo Pareschi	Gradient-based Monte Carlo methods for hyperbolic and kinetic equations	
noon-2:00pm	Lunch		
2:00pm-3:00pm	Christopher Henderson	The Boltzmann equation with large data	Zhifei Zhang
3:00pm-4:00pm	Young-Pil Choi	Quantified overdamped limit for Vlasov-Fokker-Planck equations with singular interaction forces	
4:00pm-4:30pm	Tea break		
4:30pm-5:30pm	Haitao Wang	Stability of background perturbation for Boltzmann equation	

Dec. 7th (Thu)			
Time	Speaker	Title	Host
8:30am-9:30am	Francis Filbet	A structure and asymptotic preserving scheme for the Vlasov-Poisson-Fokker-Planck model	Shi Jin
9:30am-10:00am	Tea Break		
10:00am-11:00am	Yanli Wang	Data-driven non-local reduced order modeling for radiative transfer equations	
11:00am-noon	Liu Liu	Multi-fidelity method for a class of kinetic problems with uncertainties	
noon-2:00pm	Lunch		
2:00pm-6:00pm	Free Afternoon		

