

# PKU algebra & geometry mini-forum

## 代数与几何迷你校友论坛

August 1, 2023

77201, Jingchunyuan 78, BICMR, Peking University

9:30 – 10:10

QI LU 戚鲁 (Princeton University)

*Convexity of multiplicities of filtrations on local rings*

In this talk, I will discuss some convexity properties of multiplicities of filtrations on a local ring. In particular, there is a metric on the space of filtrations, which is similar to the Darvas metric in complex geometry. The multiplicity function is convex along geodesics of this metric. As a major application, this gives a new proof of a theorem due to Xu and Zhuang on the uniqueness of normalized volume minimizers. This talk is mainly based on joint work with Harold Blum and Yuchen Liu.

10:20 – 11:00

LIU JIHAO 刘济豪 (Northwestern University)

*Minimal model program for generalized pairs*

Generalized pairs are a natural structure in birational geometry that first appeared in Kodaira's canonical bundle formula for elliptic fibrations. They were formally introduced by Birkar and Zhang in 2014 as part of their study of effective Iitaka fibrations and have since become a central topic in modern-day birational geometry. In particular, this structure have been essentially utilized in the proof of the Borisov–Alexeev–Borisov conjecture and the McKernan–Shokurov conjecture. In a series of recent works of myself, Hacon, and Xie, we have established the foundation of the minimal model program (cone theorem, contraction theorem, and the existence of flips) for generalized pairs. In this talk I will discuss these results and provide applications.

*Tea break*

11:30 – 12:10

XUE QINGYUAN 薛庆源 (University of Utah)

*Minimal Model Program in positive and mixed characteristic*

One of the fundamental goals of algebraic geometry is to classify all algebraic varieties (up to birational equivalence), which, conjectually, can be achieved by the Minimal Model Program (MMP). In characteristic 0, the theory of MMP has been developed for decades, and many important results have been proven. Recently there has also been much progress in developing the MMP in positive and mixed characteristics. In this talk I will introduce the MMP in positive and mixed characteristic, mainly focusing on that for semi-stable fourfolds. Based on joint work with Lingyao Xie.

14:00 – 14:40

GE TANGLI 葛汤立 (Princeton University)

*Some uniformity in Mordell–Lang problems*

Known as the Mordell conjecture, a nice curve of genus at least 2 over a number field contains only finitely many rational points. The classical Mordell–Lang conjecture is a generalization of this finiteness result for arbitrary varieties which can be embedded in an abelian variety. After recalling the classical results, I will introduce a rather uniform version of this finiteness result and some key ingredients in the proof. The focus will be on the geometry side. This is joint work with Ziyang Gao and Lars Kühne.

14:50 – 15:30

HU YUXUAN 胡宇瑄 (Northwestern University)

*The coherent–constructible correspondence in the relative setting*

The coherent-constructible correspondence asserts that the category of coherent sheaves on a toric variety is equivalent to the category of constructible sheaves on a real torus with certain prescribed singular supports. We will discuss this theorem with examples, after Fang–Liu–Treumann–Zaslow, and Kuwagaki. Then we will introduce the notion of principal toric fibrations, setting up the stage that allows us to describe a relative version of said theorem. Based on joint work with Pyongwon Suh.

*Tea break*

16:00 – 16:40

LI WENYUAN 李文远 (Northwestern University)

*Duality and spherical adjunction from microlocalization*

Consider sheaves on a manifold with Legendrian singular support and microlocal sheaves on the Legendrian. We study the microlocalization along the Legendrian and its left adjoint. Our result says that, when the Legendrian is a full stop or a swappable stop, these functors form a spherical adjunction, so that twists/cotwists are the monodromy/wrap-once functors. Moreover, we prove that the wrap-once functor is the inverse Serre functor when restricting to the proper subcategories. This is the sheaf theory counterpart of the spherical adjunction for cap and cup functors between Fukaya categories for Landau–Ginzburg models. We will explain how these results follow from the sheaf theory counterpart of the Sabloff duality exact sequence or the Poincaré–Lefschetz duality for Fukaya–Seidel categories. This is joint work with Christopher Kuo.