Young Mathematician Workshop on Computational and Applied Mathematics

Time: November 21, 2022
Venue: Online (Voov 401-433-350, Link: https://meeting.tencent.com/dm/0y4z168QdEB3)

The workshop aims to bring together young, energetic researchers working in applied and computational math to exchange ideas and facilitate collaborations, and to provide a hospitable platform for junior mathematicians with expertise in trending topics to present their recent works.

Schedule:

Time	Speaker	Title
8:30-9:25	Weiqi Chu (UCLA)	A mean-field opinion model on hypergraphs: From modeling to inference
9:30-10:25	Zhengyu Huang (Caltech)	Next-Generation Mathematical Methods for Science and Engineering
10:30-10:45	Tea Break	
10:45-11:40	Zhongjian Wang (UChicago)	Deep learning of multi-scale PDEs based on data generated from particle methods
2:00-5:00	Free discussion	

Talk information:

A mean-field opinion model on hypergraphs: From modeling to inference

Weiqi Chu (UCLA)

The perspectives and opinions of people change and spread through social interactions on a daily basis. In the study of opinion dynamics on networks, one often models entities as nodes and their social relationships as edges, and examines how opinions evolve as dynamical processes on networks, including graphs, hypergraphs, multi-layer networks, etc. In this talk, I will introduce a model of opinion dynamics and derive its mean-field limit, where the opinion density satisfies a kinetic equation of Kac type. We prove properties of the solution of this equation, including nonnegativity, conservativity, and steady-state convergence.

The parameters of such opinion models play a nontrivial role in shaping the dynamics. However, in reality, these parameters often can't be measured directly. In the second part of the talk, I will approach the problem from an `inverse' perspective and present how to infer the interaction kernel from limited partial observations. I will provide sufficient conditions of measurement for two scenarios, such that one is able to reconstruct the kernel uniquely. I will also provide a numerical algorithm of the inference when the data set only has a limited number of data points.

Next-Generation Mathematical Methods for Science and Engineering

Zhengyu Huang (Caltech)

In the past decades, partial differential equation (PDE)-based computational models are rapidly becoming indispensable for science and engineering. However, remarkable gaps still exist between state-of-the-art simulations and reality, and these simulations are ineffective in supporting decision-making or design under uncertainty for complex systems (e.g., climate change). To bridge the gap and fulfill challenging real-world missions, I develop data-aware computational models and practical mathematical methods to combine the exponential growth of data with complex PDE-based models.

In this talk, I will mainly focus on two important applied mathematical problems: Bayesian inference and operator learning. 1) Bayesian inference uses data to calibrate/improve models and quantify uncertainties. For large-scale science and engineering problems, challenges arise from the need for repeated evaluations of an expensive forward model, which is often given as a black box or is impractical to differentiate. Our framework, Kalman inversion, built on Kalman methodology and Fisher-Rao gradient flow, is derivative-free and generally converges in O(10) iterations. 2) Operator learning uses data to build deep learning surrogate models for PDE solving to accelerate many-query problems (e.g., design optimization). Our approach, geometry-aware Fourier neural operator, inspired by adaptive mesh motion and spectral methods, learns operators between infinite-dimensional function spaces in a resolution/discretization invariant manner. Specifically, when we learn operators between the design geometry space and the simulation solution space, our approach enables efficient engineering design optimization.

These methods we developed have been successfully applied in complex applications ranging from Mars landing supersonic parachute, bacteria-resistant catheter design, the digital twin for airfoil damage detection, and the Earth system model for climate science.

Deep learning of multi-scale PDEs based on data generated from particle methods

Zhongjian Wang (UChicago)

Solving multi-scale PDEs is difficult in high-dimensional and/or convection-dominant cases. The interacting particle methods (IPM) are shown to outperform solving PDEs directly. Examples include computing effective diffusivities, KPP front speed, and asymptotic transport properties in topological insulators. However, the particle simulation takes a long time before convergence and is lack of surrogate models for physical parameters. In this regard, we introduce the DeepParticle methods, which learn the pushforward map from arbitrary distribution to IPM-generated distribution by minimizing the Wasserstein distance. In particular, we formulate an iterative scheme to find the transport map and prove the convergence. On the application side, in addition to KPP invariant measures, our method also applies to investigate the blow-up behavior in chemotaxis models.