# Workshop on Selected Topics in Differential Geometry

**June 4-6, 2021**
**Room77201, Jingchunyuan 78, BICMR, Peking University**

### 张会春

Title: One-phase free boundary problems on Alexandrov spaces.
Abstract: In this talk, we will introduce some regularity results for variational free boundary problems on Alexandrov sapces with curvature bounded from below. It contains the Lipschitz regularity of solutions and the finite perimeter property of the free boundary. This is based on a joint work with Chung-Kwong Chan, and Xi-Ping Zhu.

### 陈小杨

Title: Rational ellipticity of Riemannian manifolds
Abstract: It was conjectured by Bott-Grove-Halperin that a compact simply connected Riemannian manifold with nonnegative sectional curvature is rationally elliptic, i.e., it has finite dimensional rational homotopy groups. We will discuss some recent progress on this conjecture.

### 夏超

Title: Stability and Morse index for minimal/CMC hypersurfaces with free boundary
Abstract: In this talk, we discuss the variational problems of area or energy functional with or without volume constraint in a compact manifold with boundary M. The stationary points are either minimal or CMC hypersurfaces with free boundary or with constant contact angle. We focus on the problem of stability and Morse index estimates, especially when M is a Euclidean ball.

### 毛井

Title: Curvature, comparison theorems and applications in spectral geometry
Abstract: In this talk, I would like to give an introduction to some comparison theorems (including my related results obtained since 2013) holding under suitable curvature constraints. Of course, their interesting applications in spectral geometry would be mentioned as well.

### 许小卫

Title: On pseudoholomorphic map between almost Hermitian manifolds
Abstract: In this talk, we use the canonical connection instead of Levi-Civita connection to study the smooth maps between almost Hermitian manifolds, especially, the pseudoholomorphic ones. By using the Bochner formulas, we obtian the $C^2$-estimate of canonical second fundamental form, Liouville type theorems of pseudoholomorphic maps, pseudoholomorphicity of pluriharmonic maps, and Simons integral inequality of pseudoholomorphic isomety. This is a joint work with C.K.Peng.

## 徐国义

Title: The lower bound of the first eigenvalues on manifolds with nonnegative Ricci curvature.

Abstract: Lichnerowicz firstly proved the sharp lower bound of the first non-zero Neumann eigenvalue on manifolds with $Rc\geq (n-1)$. Later Li-Yau developed gradient estimate to study the eigenvalue estimate, Zhong-Yang and Andrews-Clutterbuck obtained some optimal results by refined this method. Also Gromov used the Heintze-Karcher comparison theorem to prove the related eigenvalue estimates, which was developed by Buser and other people. Recent years, optimal transport method is also used to study the eigenvalue problem. We will survey those research and present our progress (joint with Haibin Wang and JieZhou) in this field.

## 吴云辉

Title: On ends of finite-volume noncompact manifolds of nonpositive curvature

Abstract: In this paper we confirm a folklore conjecture which suggests that for a complete noncompact manifold $M$ of finite volume with sectional curvature $-1 \leq K \leq 0$, if the universal cover of $M$ is a visibility manifold, then the fundamental group of each end of $M$ is almost nilpotent. Applications on the geometry and topology of noncompact nonpositively curved manifolds will be discussed. This is a joint work with Ran Ji.

## 李逸

Title: 几何流长时间存在性和曲率有界性

Abstract: 本报告主要讨论 Ricci 流、LYZ 流和 Laplacian 流的长时间存在性和曲率有界性的关系。

## 来米加

Title: Rigidity of manifolds with boundary.

Abstract: This is a survey talk　on results regarding rigidity phenonmenon for manifolds with boundary.

## 王作勤

Title: On Weyl Laws

Abstract: Weyl law is a key result in spectral geometry and has played an important role in various subjects in mathematics and physics. It was first discovered by H. Weyl in 1911 for the Dirichlet-Laplace eigenvalues of bounded regions and then extended/strengthened by many mathematicians to various general settings. In each case the Weyl law reveals relation between the semiclassical behavior of eigenvalues of certain (pseudo)differential operators and the background geometric/analytic/dynamic data. In this talk I will give a survey on this topic.

## 张世金

Title: Introduction to shrinking gradient Ricci solitons

Abstract: Shrinking gradient Ricci soliton is as generalization of Einstein metric, is also as a self-similar solution of Ricci flow and as a critical point of Perelman's entropy. In this talk, I will talk about some properties of the shrinking gradient Ricci solitons and some related problems.

**吴鹏**

Title: Complex structures on Einstein four-manifolds

Abstract: The question that when a four-manifold with a complex structure admits a compatible Einstein metric of positive scalar curvature has been answered by Tian, LeBrun, respectively. Tian classified Kahler-Einstein four-manifolds with positive scalar curvature, LeBrun classified Hermitian, Einstein four-manifolds with positive scalar curvature. In this talk we consider the inverse problem, that is, when a four-manifold with an Einstein metric of positive scalar curvature admits a compatible complex structure. We will show that if the determinant of the self-dual Weyl curvature is positive then the manifold admits a compatible complex structure.

**彦文娇**

Title: Applications of isoparametric theory

Abstract: In this talk, we will talk about our work on the applications of isoparametric foliations. For instance, Yau's conjecture on the first eigenvalues, simply connected examples for the Besse's problems, etc. Besides, we will also talk about our recent work on the Chern's conjecture. The talk is mainly based on joint work with Professor Zizhou Tang.

**胥世成**

Title: Willmore energy (resp. total squared curvature) of  submanifolds (resp. curves) on Hadamard manifold.

Abstract: We give a (non-complete) survey on the open problems and conjectures related to the Willmore energy of submanifolds and curves in a Hadamard manifold.

**刘宇航**

Title: 正截面曲率流形简介

Abstract: 我首先介绍正截面曲率流形的一些基本性质和定理（如经典的 Synge 定理，Bonnet-Myers 定理，Frenkel 定理，Wilking's connected lemma）,和著名猜想（如 Hopf 猜想）。然后我简单列举一下现在已知的正曲率流形的例子，以及相应的构造方法。最后如果时间允许的话，介绍一下 Karsten Grove's symmetry program 以及一些相关进展。