

## 会议日程

2021年5月24日上午		
9:00-9:30	<b>Opening ceremony</b>	
9:30-10:20	Fundamental Gap Estimate in Hyperbolic Space (online)	<b>韦国芳</b> (美国) 加利福尼亚大学圣塔芭芭拉分校
10:20-10:40	茶歇	
10:40-11:30	Analytic approximation and Monge-Ampère mass of plurisubharmonic singularities (online)	<b>李驰</b> (美国) 罗格斯大学
2021年5月24日下午		
14:00-14:50	Isoparametric Submanifolds and Mean Curvature Flow	<b>刘小博</b> 北京大学
15:00-15:50	Some mathematical problems of gravitational collapse in general relativity	<b>黎俊彬</b> 中山大学
15:50-16:10	茶歇	
16:10-17:00	Recent regularity results in optimal transport and free boundary problems (online)	<b>刘佳莹</b> (澳大利亚) 卧龙岗大学

2021年5月25日上午		
9:00-9:50	Topological property of hypersurfaces of low entropy (online)	<b>汪璐</b> (美国) 加州理工学院
9:50-10:30	<b>Special session+茶歇</b>	
10:30-11:20	Cohn-Vossen inequality on certain noncompact Kahler manifolds (online)	<b>刘钢</b> 华东师范大学

11:30-12:00	Equivariant min-max theory under the setting of Almgren-Pitts	王童瑞 南京大学
2021 年 5 月 25 日下午		
14:00-14:50	Positive mass theorems of ALF and ALG manifolds (online)	史宇光 北京大学
15:00-15:50	Some regularity estimates of complex Monge-Ampère equation	张希 中国科学技术大学
15:50-16:10	茶歇	
16:10-17:00	On Thurston's "geometric ideal triangulation" conjecture	葛化彬 中国人民大学

2021 年 5 月 26 日上午		
9:00-9:50	Topology and Curvature of Isoparametric Families in Spheres	唐梓洲 南开大学
10:00-10:50	Finite generation and the Kähler-Ricci soliton degeneration (online)	庄梓铨 (美国) 麻省理工学院
10:50-11:10	茶歇	
11:10-12:00	Liouville theorem for a class semilinear elliptic equation on Heisenberg group	麻希南 中国科技大学
2021 年 5 月 26 日下午		
自由讨论		

2021 年 5 月 27 日上午		
-------------------	--	--

9:00-9:50	Gradient and Eigenvalue Estimates on the canonical bundle of Kähler manifolds (online)	张旗 (美国) 加州大学河滨分校
9:50-10:10	茶歇	
10:10-11:00	Existence of deformed Hermitian-Yang Mills metric	韩小利 清华大学
11:10-12:00	Khovanskii-Teissier type inequalities and Hodge-Riemann bilinear relations	张雅山 湖南大学
<b>2021 年 5 月 27 日下午</b>		
14:00-14:50	PDE approach to anisotropic Minkowski inequality for non-convex domains (online)	夏超 厦门大学
15:00-15:30	Boundary regularity for the oblique problem of the Monge-Ampère equation (online)	涂绪山 清华大学
15:30-15:50	茶歇	
15:50-16:40	Convex hypersurfaces in the sphere with prescribed Gauss curvature (online)	李奇睿 浙江大学
16:50-17:20	Moser-Trudinger type inequality for plurisubharmonic functions and complex Monge Ampere equations	王嘉项 浙江大学

<b>2021 年 5 月 28 日上午</b>		
9:00-9:50	On the Holder estimate of Kahler-Ricci flow (online)	楚健春 (美国) 西北大学
9:50-10:10	茶歇	
10:10-11:00	Isolated singularities of solutions to the Yamabe equation with non-conformally flat metrics (online)	熊金钢 北京师范大学
11:10-12:00	The optimal exponent of certain Moser--Trudinger type inequality	张科伟 北京大学

2021 年 5 月 28 日下午		
14:00-14:50	The asymptotic geometry of 4D steady Ricci solitons	邓宇星 北京理工大学
15:00-15:30	Convergence of Kähler Ricci flow near a Kähler Ricci soliton	张亮 北京大学
15:30-15:50	茶歇	
15:50-16:20	Topology of surfaces with finite Willmore energy	周杰 清华大学
16:30-17:00	Sub-Laplacian comparison theorem and Weitzenböck type formula on Riemannian foliations	母贵 复旦大学

## 50 分钟报告

### On the Holder estimate of Kahler-Ricci flow

楚健春(online)

In this talk, I will discuss the Holder estimate of the Kahler-Ricci flow on compact Kahler manifolds with semi-ample canonical line bundle. I will introduce the recent work with Man-Chun Lee where we establish the spatial Holder estimate along the Kahler-Ricci flow by adapting the method in the work of Hein-Tosatti on collapsing Calabi-Yau metrics.

### The asymptotic geometry of 4D steady Ricci solitons

邓宇星

The asymptotic geometry is important for the classification of Ricci solitons. In this talk, we will classify the asymptotic geometry of 4D steady gradient Ricci solitons whose Ricci curvature is nonnegative outside a compact set. As an application, we will show such steady Kahler-Ricci solitons must be Ricci-flat.

### On Thurston's "geometric ideal triangulation" conjecture

葛化彬

Using combinatorial Ricci flow methods, we shall prove the following theorem: Let  $M$  be a compact 3-manifold with boundary consisting of surfaces of genus at least 2. If  $M$  admits an ideal triangulation with valence at least 10 at all edges, then there exists a unique hyperbolic

metric on  $M$  with totally geodesic boundary under which the ideal triangulation is geometric. For a special class of 3-manifolds, the theorem affirms a folklore conjecture which exists for almost 40 years under not so strong assumptions. This is based on joint work with Ke Feng and Bobo Hua.

### **Existence of deformed Hermitian-Yang Mills metric**

韩小利

First I will introduce the equation of the deformed Hermitian-Yang Mills metric and some existence results of this equation under some assumptions. I also give some results about this equation with Dirichlet or Neuman boundary value. Then I will introduce the heat equation of the deformed Hermitian-Yang Mills metric and some long time existence and convergence of the heat flow.

### **Analytic approximation and Monge-Ampère mass of plurisubharmonic singularities**

李驰([online](#))

I will construct examples of plurisubharmonic functions with isolated singularities, whose residual Monge-Ampère mass can not be approximated by mass of canonical analytic approximations obtained via multiplier ideals. This answers negatively a conjecture of Demailly, and shows that residual Monge-Ampère mass is not a valuative invariant of plurisubharmonic singularity. These examples also suggest a new characterization of  $\omega$ -psh functions of full mass on projective manifolds, which has been proved recently by Thai and Vu.

### **Some mathematical problems of gravitational collapse in general relativity**

黎俊彬

I will give a brief review on the mathematical theories of gravitational collapse in general relativity, and talk about some problems and recent developments, including my works on the formation of trapped surfaces, black holes and instability of naked singularities.

### **Convex hypersurfaces in the sphere with prescribed Gauss curvature**

李奇睿([online](#))

Convex hypersurfaces in the sphere with prescribed Gauss curvature was studied by many authors under the assumption of barriers or certain symmetry. We show the existence of solutions to the problem without such assumption. Multiple solutions are also discussed.

### **Cohn-Vossen inequality on certain noncompact Kahler manifolds**

刘钢([online](#))

We generalize the Cohn-Vossen inequality to certain noncompact Kahler manifolds. This is related to a conjecture of Yau.

### **Recent regularity results in optimal transport and free boundary problems.**

刘佳堃(online)

In this talk, we first give a brief introduction to the optimal transport problem, and then introduce some of our recent results on global regularity theory. Last, we introduce the free boundary problem in optimal partial transport, and also our recent results on the regularity of free boundaries. These are joint works with Shibing Chen (USTC) and Xu-Jia Wang (ANU).

### **Isoparametric Submanifolds and Mean Curvature Flow**

刘小博

Ancient solutions are important in studying singularities of mean curvature flows (MCF). So far most rigidity results about ancient solutions are modeled on shrinking spheres or spherical caps. In this talk, I will describe the behavior of MCF for a class of submanifolds, called isoparametric submanifolds, which have more complicated topological type. We can show that all such solutions are in fact ancient solutions, i.e. they exist for all time which goes to negative infinity. Similar results also hold for MCF of regular leaves of polar foliations in simply connected symmetric spaces with non-negative curvature. I will also describe our conjectures proposed together with Terng on rigidity of ancient solutions to MCF for hypersurfaces in spheres. These conjectures are closely related to Chern's conjecture for minimal hypersurfaces in spheres. This talk is based on joint works with Chuu-Lian Terng and Marco Radeschi.

### **Liouville theorem for a class semilinear elliptic equation on Heisenberg group**

麻希南

We obtain a Liouville type theorem to the classical semilinear subcritical elliptic equation on Heisenberg group. The soul of the proof is an a priori integral estimate, which is deduced from a generalized differential identity found by Jerison and Lee in 1988. We also get a pointwise estimate of near the isolated singularity. This is a joint work with Prof. Qianzhong Ou.

### **Positive mass theorems of ALF and ALG manifolds**

史宇光(online)

In this talk, we will discuss positive mass theorems for ALF and ALG manifolds with dimensions no greater than 7. Different from the compatibility condition for spin structure in Theorem 2 of V. Minerbe's paper A mass for ALF manifolds, Comm. Math. Phys. 289 (2009), no. 3, 925–955, we show that some type of simple topological conditions of manifolds is enough to guarantee the nonnegativity of the mass. As in the asymptotically flat case, we reduce the desired positive mass theorems to those ones concerning non-existence of positive scalar curvature metrics on closed manifolds coming from generalize surgery to  $n$ -torus. Finally, we investigate certain fill-in problems and obtain an optimal bound for total mean curvature of admissible fill-ins for flat product 2-torus.

This talk is based on the paper joint with my Ph.D. students Peng Liu and Jintian Zhu, here is the

link of the paper:  
<http://arxiv.org/abs/2103.11289>

## **Topology and Curvature of Isoparametric Families in Spheres**

唐梓洲

An isoparametric family in the unit sphere consists of parallel isoparametric hypersurfaces and their two focal submanifolds. The talk has two parts. The first part investigates topology of the isoparametric families, namely the homotopy, homeomorphism, or diffeomorphism types, parallelizability, as well as the LS category. This part extends substantially the results of Q.M.Wang in 1988 JDG. The second part is concerned with their curvatures, we determine when they have non-negative sectional curvatures or positive Ricci curvatures with the induced metric. The talk is based on the joint work with Chao Qian and Wenjiao Yan.

## **Topological property of hypersurfaces of low entropy**

汪璐([online](#))

Colding and Minicozzi introduced a notion of entropy for hypersurfaces which is given by the supremum of Gaussian integrals with varying centers and scales. In this talk, I will survey some recent results on topological properties of hypersurfaces of low entropy. The main tool is certain weak mean curvature flows.

## **Fundamental Gap Estimate in Hyperbolic Space**

韦国芳([online](#))

In their celebrated work, B. Andrews and J. Clutterbuck proved the fundamental gap conjecture that difference of first two eigenvalues of the Laplacian with Dirichlet boundary condition on convex domain with diameter  $D$  in the Euclidean space is greater than or equal to  $3\pi^2/D^2$ . In several joint works with X. Dai, Z. He, S. Seto, L. Wang (in various subsets) the estimate is generalized, showing the same lower bound holds for convex domains in the unit sphere. In sharp contrast, in recent joint work with T. Bourni, J. Clutterbuck, X. Nguyen, A. Stancu and V. Wheeler, we prove that the product of the fundamental gap with the square of the diameter can be arbitrarily small for convex domains of any diameter in hyperbolic space. Very recently, joint with X. Nguyen, A. Stancu, we show even for horoconvex domains in the hyperbolic space, the product of their fundamental gap with the square of their diameter has no positive lower bound.

## **PDE approach to anisotropic Minkowski inequality for non-convex domains**

夏超([online](#))

In this talk, we discuss the anisotropic Minkowski inequality, which is an isoperimetric type inequality between anisotropic mean curvature integral and anisotropic area, for star-shaped  $F$ -mean convex domains or outward  $F$ -minimizing domains. Our method is based on the inverse anisotropic mean curvature flow and the anisotropic capacity, respectively. Part of the work is

joint with Dr. Jiabin Yin.

### **Isolated singularities of solutions to the Yamabe equation with non-conformally flat metrics**

熊金钢(online)

The isolated singularities of solutions to the Yamabe equation with conformally flat background metric were completely classified in the celebrated paper of Caffarelli-Gidas-Spruck 1989. When the background metrics are not conformally flat, Marques 2008 and Xiong-Zhang 2020 solved the problem up to dimension 6. In this talk, I will report our recent work which settles the problem in higher dimensions. This is joint with Z. C. Han & L. Zhang.

### **The optimal exponent of certain Moser--Trudinger type inequality**

张科伟

In this talk I will show that Fujita-Odaka's delta invariant coincides with the optimal exponent of certain Moser--Trudinger type inequality on polarized manifolds. As a consequence we obtain a uniform Yau-Tian-Donaldson theorem for the existence of twisted Kahler-Einstein metrics, which generalizes the work of Berman--Boucksom--Jonsson to the non-Fano setting. A new computable criterion for the existence of constant scalar curvature metrics will also be given.

### **Gradient and Eigenvalue Estimates on the canonical bundle of Kähler manifolds**

张旗(online)

In this joint work with Zhiqin Lu and Zhu Meng, we prove certain gradient and eigenvalue estimates, as well as the heat kernel estimates, for the Hodge Laplacian on  $(m,0)$  forms, i.e., sections of the canonical bundle of Kähler manifolds, where  $m$  is the complex dimension of the manifold. Instead of the usual dependence on curvature tensor, our condition depends only on the Ricci curvature bound. The proof is based on a new Bochner type formula. forms, which involves only the Ricci curvature and the gradient of the scalar curvature.

### **Some regularity estimates of complex Monge-Ampère equation**

张希

The complex Monge-Ampère equation has been the subject of intensive studies in the past forty years, because of its important application in complex geometry.

In this talk, I will introduce our recent work on regularity estimates of the complex Monge-Ampère equation and its applications in Kähler geometry and Sasakian geometry. These work are joint with P.F.Guan, C.Li, J.Y.Li, J.W.Liu and C.J.Zhang.

### **Khovanskii-Teissier type inequalities and Hodge-Riemann bilinear relations**

张雅山

Khovanskii-Teissier inequalities in complex geometry are profound analogs of Alexandrov-Fenchel inequalities in convex geometry. In this talk, we shall present some new Khovanskii-Teissier type inequalities in certain mixed and degenerate settings, whose proofs are basing on developing the corresponding new Hodge-Riemann bilinear relations on Kaehler manifolds.

### **Finite generation and the Kähler-Ricci soliton degeneration**

庄梓铨(online)

By the Hamilton-Tian conjecture on the limit behavior of Kähler-Ricci flows, every complex Fano manifold degenerates to a Fano variety that has a Kähler-Ricci soliton. Moreover, the degeneration limit is unique by a recent result of Han and Li. In this talk, I'll discuss the algebro-geometric analogue of these statements and show that the Kähler-Ricci soliton degeneration exists for every possibly singular Fano variety. This relies on a birational geometric result on the finite generation of certain graded rings that are naturally associated to the Fano varieties. Based on recent joint work with Harold Blum, Yuchen Liu and Chenyang Xu.

### **30 分钟报告**

### **Sub-Laplacian comparison theorem and Weitzenböck type formula on Riemannian foliations**

母贵

In this talk, we mainly study the following:

(1) Based on Bott connection, by introducing a kind of novel metric connection adapted to Riemannian foliations, we develop a variational theory of geodesics for the canonical variation of the metric of Riemannian foliations. Under some geometric conditions, we establish horizontal and vertical comparison theorems for Riemannian distance function. As a corollary, we find the Bonnet-Myers type theorem on the Riemannian foliation. By using sub-Laplacian comparison theorem, we obtain the corresponding generalized maximum principle. Moreover, we give some applications of the generalized maximum principle.

(2) By introducing the other kind of affine connection, we derive Weitzenböck type formula and Bochner type formula of the sub-Laplacian on Riemannian foliations. Under some geometric conditions, we could generalize Baudoin-Garofalo type curvature dimensional inequalities in the case of totally geodesic Riemannian foliations to arbitrary Riemannian foliations. This shows Carnot groups meet our generalized curvature dimensional inequalities.

This is a joint work with Y. X. Dong.

### **Boundary regularity for the oblique problem of the Monge-Ampère equation**

涂绪山(online)

In this talk we shall talk about our recent works on the boundary regularity for the oblique problem of the Monge-Ampère equation. For the two-dimensional case, we obtain the Schauder

estimate and the  $W^{2,1+\epsilon}$  estimate. When the space dimension  $n \geq 3$ , we show that the Schauder estimate still holds locally at the boundary point  $x_0$  if the viscosity convex solution is bounded quadratic growth in the tangent direction at point  $x_0$ .

### **Moser-Trudinger type inequality for plurisubharmonic functions and complex Monge Ampere equations**

王嘉项

In this talk I wish to share a proof of Moser-Trudinger type inequality on pseudo-convex domains, which can derive a PDE approach to the a priori estimates for complex Monge Ampere equations. These are adjoint works with Prof. Xu-Jia Wang and Prof. Bin Zhou.

### **Equivariant min-max theory under the setting of Almgren-Pitts**

王童瑞

The Almgren-Pitts min-max theory was first proposed in the 1960s to find minimal hypersurfaces in compact manifolds which had many excellent applications in recent years. In this talk, I will introduce an equivariant version of the Almgren-Pitts min-max theory which can construct  $G$ -invariant minimal hypersurfaces in a closed manifold  $M$  with a compact Lie group  $G$  acting as isometries. Additionally, the growth of  $p$ -width build by Gromov and Guth can also be generalized into an equivariant version. With its help, we further generalize the work of Marques-Neves and obtain infinite many  $G$ -invariant minimal hypersurfaces on closed manifolds with positive Ricci curvature.

### **Convergence of Kähler Ricci flow near a Kähler Ricci soliton**

张亮

We will derive a new version of formula for the second variation of Perelman's entropy in the space of Kähler-metric near a Kähler-Ricci soliton. Then by establishing Lojasiewicz's typed inequality and local maximality for Perelman's entropy, we will study the convergence of Kähler Ricci flow near a Kähler Ricci soliton. And we may give some applications for the convergence Theorem.

### **Topology of surfaces with finite Willmore energy**

周杰

In this presentation, we care about the critical case of Allard's regularity theorem and its applications. By combining Allard's original method with Reifenberg's topological disk theorem, we get a critical Allard-Reifenberg type regularity theorem. As a main result, we get the topological finiteness for a class of properly immersed surfaces in  $R^n$  with finite Willmore energy. Especially, we prove the removability of isolated singularity of multiplicity one surfaces with finite Willmore energy and a uniqueness theorem of the catenoid under no a priori topological finiteness or geometric finiteness assumption.