A DERIVATION OF THE VLASOV EQUATION FROM A QUANTUM MANY-BODY FERMIONIC SYSTEM WITH SINGULAR INTERACTION

JACKY CHONG
DEPARTMENT OF MATHEMATICS
THE UNIVERSITY OF TEXAS, AUSTIN, USA

1. Abstract

We consider the combined mean-field and semiclassical limit for a system of the N interacting Fermions in the case of singular potentials. We prove the uniformly in the Planck constant h propagation of regularity for the Hartree–Fock equation with singular pair interaction potentials of the form $|x-y|^{-a}$, including the Coulomb interaction. Using these estimates, we obtain quantitative bounds on the distance between solutions of the Schrödinger equation and solutions of Hartree–Fock and Vlasov equations in Schatten norms. For $a \in (0, 1/2)$, we obtain local-in-time results when $N^{-1/2} \ll h \leq N^{-1/3}$. In particular, it leads to the derivation of the Vlasov equation with singular potentials. For $a \in (1/2, 1]$, our results hold only on a small time scale $t \sim h^{a-1/2}$, or with a N dependent cutoff. This is a joint work with Laurent Lafleche[1] and Chiara Saffirio[2]; see `https://arxiv.org/abs/2103.10946`.

Department of Mathematics, The University of Texas at Austin
Email address: `jwchong@math.utexas.edu`

[1]Department of Mathematics, The University of Texas at Austin, Austin, TX 78712, USA, `lafleche@math.utexas.edu`.
[2]Department of Mathematics and Computer Science, University of Basel, 4051 Basel, Switzerland, `chiara.saffirio@unibas.ch`.