Convergence rates in homogenization of parabolic systems with locally periodic coefficients

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Abstract: This talk mainly concerns with the quantitative homogenization of second-order parabolic systems with time-dependent locally periodic coefficients in $C^{1,1}$ cylinders, i.e.,

$$\begin{cases} \partial_t u_\varepsilon - \operatorname{div}(A(x,t;\frac{x}{\varepsilon},\frac{t}{\varepsilon^2})\nabla u_\varepsilon) = f & \text{in } \Omega\times(0,T), \\ u_\varepsilon = g & \text{on } \partial\Omega\times(0,T), \\ u_\varepsilon = h & \text{on } \Omega\times\{t=0\}, \end{cases}$$

where Ω is a bounded $C^{1,1}$ domain in $\mathbb{R}^d$, the matrix $A(x,t;y,\tau)$ defined on $\Omega\times(0,T)\times\mathbb{R}^{d+1}$ is bounded, elliptic and 1-periodic in (y,τ), and $\varepsilon>0$ is a parameter.

Under nearly minimal smoothness assumptions on A which indicate the 1st order differentiability in x and $\frac{1}{2}+$ order differentiability in t, the sharp-order scale-invariant convergence rate to some u_0 is established, i.e.,

$$\|u_\varepsilon - u_0\|_{L^2(0,T;L^{\frac{2d}{d-1}}(\Omega))} \le C\varepsilon\Big\{\|\nabla u_0\|_{L^2(0,T;W^{1,\frac{2d}{d+1}}(\Omega))} + \|\partial_t u_0\|_{L^2(0,T;L^{\frac{2d}{d+1}}(\Omega))}\Big\}.$$

To do this, we employ fractional derivatives on intervals to build several almost optimal estimates for the macroscopic smoothing operator, and derive a new estimate for the integrals on temporal boundary layers. This extends the previous work [Y. Xu and W. Niu, Comm. Partial Differential Equations, 2020] about elliptic systems with stratified structure.