Geometric structures and substructures on Fano manifolds and their relationship with Kähler geometry

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Abstract In a series of articles with Jun-Muk Hwang starting from the late 1990s, we introduced a geometric theory of uniruled projective manifolds based on the variety of minimal rational tangents (VMRT), i.e., the collection of tangents to minimal rational curves on a uniruled projective manifold $(X, \mathcal{K})$ equipped with a minimal rational component. This theory provides differential-geometric tools for the study of uniruled projective manifolds, especially Fano manifolds of Picard number 1. Associated to $(X, \mathcal{K})$ is the fibered space $\pi : \mathcal{C}(X) \to X$ of VMRTs, which we will call the VMRT structure on $(X, \mathcal{K})$. More recently, with Jaehyun Hong and Yunxin Zhang we have started the study of germs of complex submanifolds S on uniruled projective manifolds inheriting geometric substructures obtained from intersections of VMRTs with tangent subspaces, giving rise to sub-VMRT structures $\varpi : \mathcal{C}(S) \to S$, $\mathcal{C}(S) := \mathcal{C}(X) \cap \mathbb{P}T(S)$. Central to the study of VMRT and sub-VMRT structures are various types of recognition problems, i.e., problems of characterizing special types of Fano manifolds of Picard number 1 or special uniruled projective subvarieties on them in terms of VMRT and sub-VMRTs. We will discuss some basic results on VMRT and sub-VMRT structures and relate these results to the study of holomorphic isometries between bounded symmetric domains. Especially, we will show how examples of nonstandard holomorphic isometric embeddings of the complex unit ball into irreducible bounded symmetric domains of rank ≥ 2 can be constructed using VMRTs and illustrate how uniqueness results can be proven for such maps in certain cases. The latter proof exploits the notion of parallel transport (holonomy), a notion of fundamental importance both in Kähler geometry and in the study of sub-VMRT structures.