Joint Power and Admission Control

Ya-Feng Liu

State Key Laboratory of Scientific/Engineering Computing,
Institute of Computational Mathematics and Scientific/Engineering Computing,
Academy of Mathematics and Systems Science,
Chinese Academy of Sciences, Beijing, China
Email: yafliu@lsec.cc.ac.cn

Joint Work with Yu-Hong Dai, Zhi-Quan Luo, Shiqian Ma, Enbin Song, and Mingyi Hong

2013 PKU Workshop on Optimization and Data Sciences

Optimization for Communication Networks

- Complexity issues of optimization problems
  - highly nonlinear
  - nonconvex
  - special structure

Optimization for Communication Networks

- Complexity issues of optimization problems
  - highly nonlinear
  - nonconvex
  - special structure
- Algorithm design by using special structures

Optimization for Communication Networks

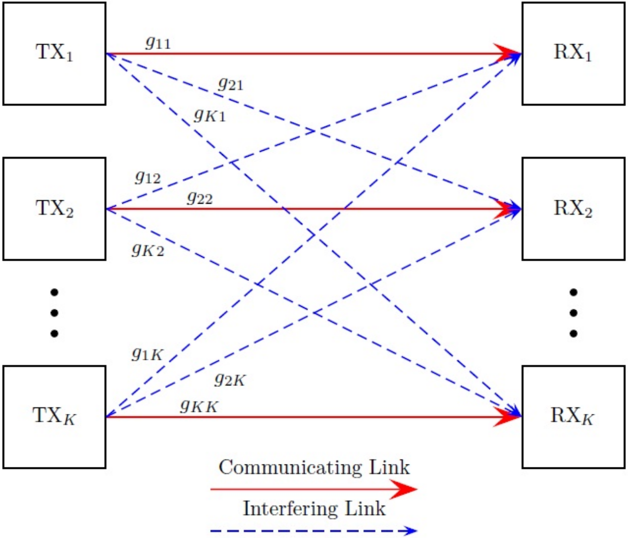
- Complexity issues of optimization problems
  - highly nonlinear
  - nonconvex
  - special structure
- Algorithm design by using special structures
  - convergence
  - optimality
  - efficiency
  - distributed implementation
  - data uncertainty

Optimization for Communication Networks

- Complexity issues of optimization problems
  - highly nonlinear
  - nonconvex
  - special structure
- Algorithm design by using special structures
  - convergence
  - optimality
  - efficiency
  - distributed implementation
  - data uncertainty
- Challenging but interesting

PROBLEM FORMULATION

K-Link SISO Interference Channel



Mathematical Model

- Consider a K-link (transmitter-receiver pair) single-input single-output (SISO) interference channel:
  - g_{kj} represents the channel gain from transmitter j to receiver k
  - η_k denotes the noise power at receiver k
  - p_k is the transmitted power at transmitter k
  - the received power at receiver k is given by

$$\underbrace{g_{kk}p_k}_{\text{desired power}} + \underbrace{\sum_{j \neq k} g_{kj}p_j + \eta_k}_{\text{interference plus noise power}}$$

  - signal to interference plus noise ratio (SINR) value at receiver k is

$$\text{SINR}_k = \frac{g_{kk}p_k}{\sum_{j \neq k} g_{kj}p_j + \eta_k}$$

Power Control

- Power control problem [Foschini-Miljanic, 1993]:

$$\begin{aligned} \min \quad & \mathbf{e}^T \mathbf{p} \\ \text{s.t.} \quad & \text{SINR}_k \geq \gamma_k, \ k \in \mathcal{K} \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}} \end{aligned}$$

  - $\mathbf{e}$: the vector with all components being one
  - γ_k : SINR target of link k
  - $\mathcal{K} = \{1, 2, \cdots, K\}$
  - $\mathbf{p} = (p_1, p_2, \cdots, p_K)^T$
  - $\bar{p}_k$ is the power budget at transmitter k
  - $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_K)^T$

Joint Power and Admission Control (JPAC)

- A long-standing issue associated with power control is that the problem often becomes infeasible.
- The admission control is necessary to determine the connections to be removed.
- In this presentation, we consider the JPAC problem, where
  - the admitted links should be satisfied with their required SINR targets
  - the number of admitted (removed) links should be maximized (minimized)
  - the total transmission power to support the admitted links should be minimized

Two-Stage Formulation

- A two-stage optimization problem:
  - Maximizes the number of admitted links (with prescribed SINR targets):

$$\begin{aligned} \max_{\mathbf{p},\,\mathcal{S}} \quad & |\mathcal{S}| \\ \text{s.t.} \quad & \text{SINR}_k \geq \gamma_k, \; k \in \mathcal{S} \subseteq \mathcal{K} \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}} \end{aligned} \tag{1}$$

  - we use $\mathcal{S}^*$ to denote the maximum admissible set for problem (1), and $\mathcal{S}^*$ might not be unique
  - Minimizes the total transmission power required to support the admitted links:

$$\begin{aligned} \min \quad & \textstyle\sum_{k \in \mathcal{S}^*} p_k \\ \text{s.t.} \quad & \text{SINR}_k \geq \gamma_k, \; k \in \mathcal{S}^* \\ & 0 \leq p_k \leq \bar{p}_k, \; k \in \mathcal{S}^* \end{aligned} \tag{2}$$

Complexity Analysis

- Power control problem (2) is feasible, and can be efficiently solved in a distributed fashion [Foschini-Miljanic, 1993]
- However, admission control problem (1) of finding the maximum admissible set $\mathcal{S}^*$ is NP-hard [Mitliagkas-Sidiropoulos-Swami, 2011; Andersin-Rosberg-Zander, 1996]
- The complexity result motivates us to develop heuristic algorithms for the JPAC problem.

Existing Algorithms

- Removal-based algorithms
  - update the power, and check whether all links in the network can be supported
  - if yes, terminate the algorithm
  - if not, remove one link from the network, and update the power again

Existing Algorithms

- Removal-based algorithms
  - update the power, and check whether all links in the network can be supported
  - if yes, terminate the algorithm
  - if not, remove one link from the network, and update the power again
- Three key steps
  - power update
  - feasibility check
  - link removal

Our Contribution

- A new LPD algorithm (Part I)
  - ℓ_0-minimization reformulation
  - new LP approximation
  - easy-to-check necessary condition
- Some extensions (Part II)
  - non-convex approximations
  - chance SINR constraints
  - distributed implementation (will be skipped)

PART I

Normalized Channel

- Two equivalent equations:
  - power constraint: $0 \leq p_k \leq \bar{p}_k \Leftrightarrow 0 \leq x_k := \frac{p_k}{\bar{p}_k} \leq 1$
  - SINR constraint: $\frac{g_{kk}p_k}{\sum_{j\neq k} g_{kj}p_j + \eta_k} \geq \gamma_k \Leftrightarrow \frac{1x_k}{\sum_{j\neq k} \frac{\gamma_k g_{kj}\bar{p}_j}{g_{kk}\bar{p}_k} x_j + \frac{\gamma_k \eta_k}{g_{kk}\bar{p}_k}} \geq 1$
- Normalized channel:
  - noise vector $\mathbf{b} = \left(\frac{\gamma_1\eta_1}{g_{11}\bar{p}_1}, \frac{\gamma_2\eta_2}{g_{22}\bar{p}_2}, \cdots, \frac{\gamma_K\eta_K}{g_{KK}\bar{p}_K}\right)^T > \mathbf{0}$
  - power allocation vector $\mathbf{x} = \left(\frac{p_1}{\bar{p}_1}, \frac{p_2}{\bar{p}_2}, \cdots, \frac{p_K}{\bar{p}_K}\right)$
  - channel gain matrix $\mathbf{A}$ with its (k, j)-th entry

$$a_{kj} = \begin{cases} -\frac{\gamma_k g_{kj}\bar{p}_j}{g_{kk}\bar{p}_k}, & \text{if } k \neq j; \\ 1, & \text{if } k = j. \end{cases}$$

Normalized Channel

With this normalization:

- Focus on $\mathbf{A}$ and $\mathbf{b}$
- $\text{SINR}_k \geq \gamma_k \Leftrightarrow [\mathbf{Ax} - \mathbf{b}]_k \geq 0$

ℓ_0-Minimization Reformulation

Theorem (L.-Dai-Luo, 2013)

The two-stage JPAC problem can be equivalently reformulated as

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_0 + \alpha\, \bar{\mathbf{p}}^T \mathbf{x} \\ s.t. \quad & \mathbf{0} \le \mathbf{x} \le \mathbf{e} \end{aligned} \tag{3}$$

where

$$0 < \alpha < \alpha_1 := 1/\bar{\mathbf{p}}^T \mathbf{e}.$$

- problem (3) can find the maximum admissible set $\mathcal{S}^*$ and at the same time minimize the total required transmission power to support the links in $\mathcal{S}^*$
- there might be more than one maximum admissible set, problem (3) is capable of picking the one with *minimum* total transmission power ☺

Linear Programming Relaxation

- Consider its ℓ_1-convex relaxation

$$\begin{array}{ll}\min\limits_{\mathbf{x}} & \|\mathbf{b}-\mathbf{A}\mathbf{x}\|_1+\alpha\,\bar{\mathbf{p}}^T\mathbf{x}\\ \text{s.t.} & \mathbf{0}\le\mathbf{x}\le\mathbf{e}\end{array}$$

- We further show ℓ_1-minimization problem is equivalent to the following linear program (LP)

$$\begin{array}{ll}\min\limits_{\mathbf{x}} & \mathbf{e}^T(\mathbf{b}-\mathbf{A}\mathbf{x})+\alpha\,\bar{\mathbf{p}}^T\mathbf{x}\\ \text{s.t.} & \mathbf{b}-\mathbf{A}\mathbf{x}\ge\mathbf{0}\\ & \mathbf{0}\le\mathbf{x}\le\mathbf{e}\end{array} \tag{4}$$

  - the quantity, $\mathbf{x}_k^e=[\mathbf{b}-\mathbf{A}\mathbf{x}]_k$, measures the excess transmission power
  - LP (4) actually minimizes a weighted sum of the total excess transmission power and the total real transmission power

Role of LP Relaxation I

- Power control
  - the optimal power allocation is given by

$$p_k = \bar{p}_k x_k, \ \ k \in \mathcal{K},$$

    where $\mathbf{x}$ is the solution of LP (4)

- Feasibility check
  - by solving LP (4) with an appropriate α, we know whether all links in the network can be simultaneously supported or not

Role of LP Relaxation II

- Link removal
  - having obtained the solution of LP (4), we can use the same idea in [Mitliagkas-Sidiropoulos-Swami, 2011], i.e., drop link k_0 with

$$k_0 = \arg\max_{k \in \mathcal{K}} \left\{ \sum_{j \neq k} |a_{jk}| x_k^e + \sum_{j \neq k} |a_{kj}| x_j^e \right\} \tag{5}$$

  - the above removal strategy can be rewritten as

$$\sum_{j \neq k} |a_{jk}| x_k^e + \sum_{j \neq k} |a_{kj}| x_j^e = \sum_{j \neq k} \frac{\gamma_j}{g_{jj} \bar{p}_j} g_{jk} p_k^e + \sum_{j \neq k} \frac{\gamma_k}{g_{kk} \bar{p}_k} g_{kj} p_j^e$$

  - different from the removal scheme in [Mitliagkas-Sidiropoulos-Swami, 2011]

$$k_0 = \arg\max_{k \in \mathcal{K}} \left\{ \sum_{j \neq k} g_{jk} p_k^e + \sum_{j \neq k} g_{kj} p_j^e \right\} \tag{6}$$

A Necessary Condition

- Define $\boldsymbol{\mu} = \mathbf{A}^T \mathbf{e}, \boldsymbol{\mu}_+ = \max\{\boldsymbol{\mu}, \mathbf{0}\}$, and $\boldsymbol{\mu}_- = \max\{-\boldsymbol{\mu}, \mathbf{0}\}$
- An easy-to-check necessary condition

$$\boldsymbol{\mu}_+^T \mathbf{e} - (\boldsymbol{\mu}_- + \mathbf{e})^T \mathbf{b} \geq 0$$

- Remove the link k_0 according to

$$k_0 = \arg\max_{k \in \mathcal{K}} \left\{ \sum_{j \neq k} |a_{kj}| + \sum_{j \neq k} |a_{jk}| + b_k \right\} \tag{7}$$

- Multi scales to accelerate the algorithm (especially for strong interference channels)

A New LPD (NLPD) Algorithm [L.-Dai-Luo, 2013]

A New Linear Programming Deflation Algorithm

Step 1. Initialization: Input data $(\mathbf{A}, \mathbf{b}, \bar{\mathbf{p}})$ and set $\mathcal{S} = \mathcal{K}$.

Step 2. Preprocessing: remove link k_0 successively according to (7) until the necessary condition holds true.

Step 3. Power control: Solve linear program (4); check whether all links are supported: if yes, go to **Step 5**; else go to **Step 4**.

Step 4. Admission control: Remove link k_0 according to (5), set $\mathcal{S} = \mathcal{S} \setminus \{k_0\}$, and go to **Step 3**.

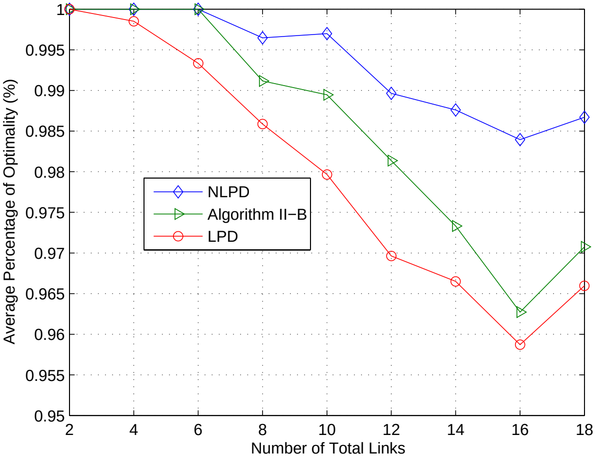
Step 5. Postprocessing: Check the removed links for possible admission.

Numerical Simulations

- The same channel parameters[1] are generated as in [Mitliagkas-Sidiropoulos-Swami, 2011]
- Compare the proposed NLPD algorithm with other algorithms including
  - LPD algorithm [Mitliagkas-Sidiropoulos-Swami, 2011]
  - Algorithm II-B [Mahdavi-Doost-Ebrahimi-Khandani, 2010]
  - "brute force" search (enumeration)
- Comparison criteria
  - number of admitted links
  - total transmission power
  - executed CPU time
- All figures report the average results for 200 Monte-Carlo runs.

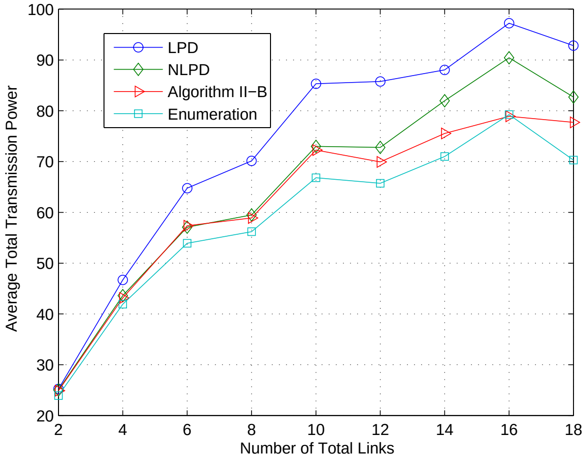
[1] Thanks Professor N. D. Sidiropoulos for his help in numerical simulations.

Average Percentage of Optimality



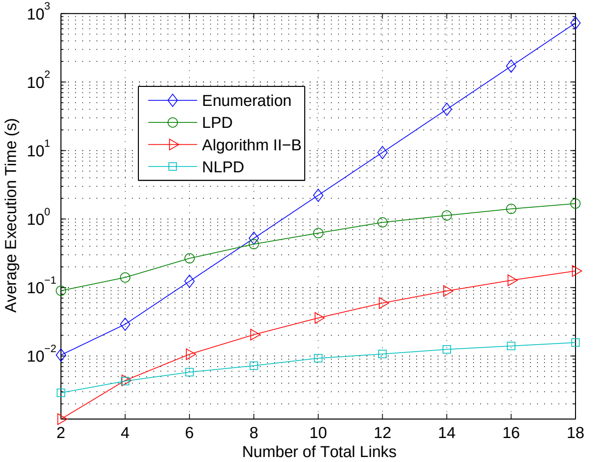
Average percentage of global optimality versus the number of total links.

Average Total Transmission Power



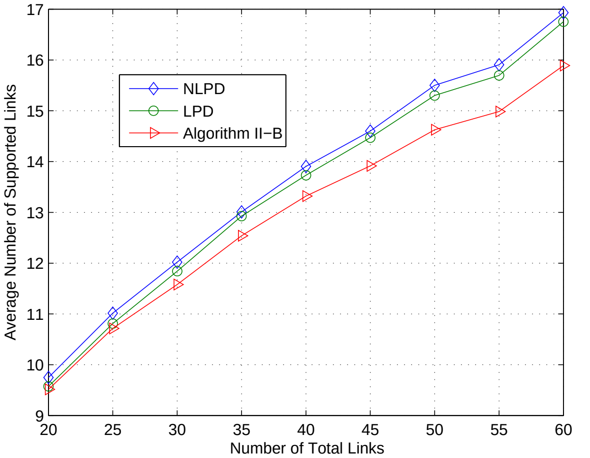
Average total transmission power versus the number of total links.

Average Execution Time



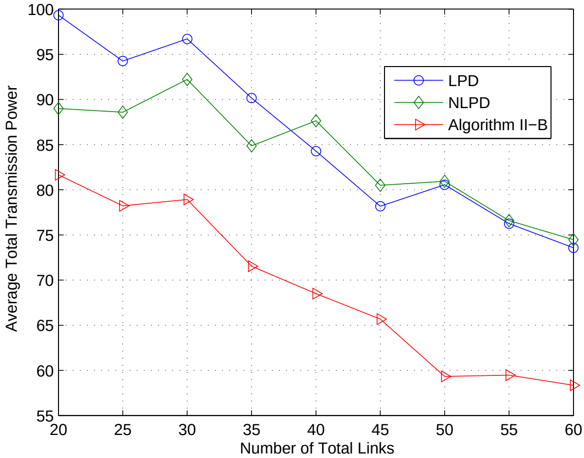
Average execution time (in seconds) versus the number of total links.

Average Number of Supported Links



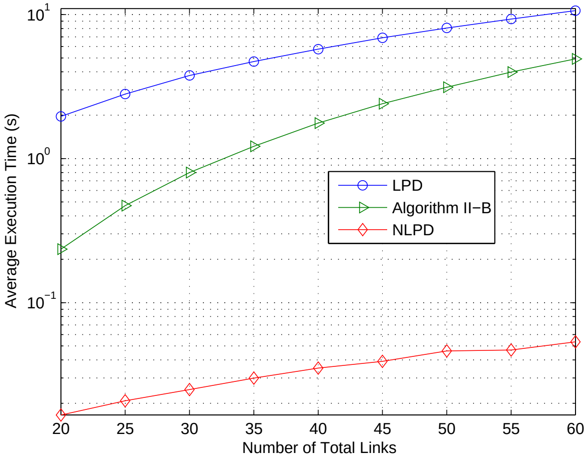
Average number of supported links versus the number of total links.

Average Total Transmission Power



Average total transmission power versus the number of total links.

Average Execution Time



Average execution time (in seconds) versus the number of total links.

PART II

Non-Convex Approximation

- L_q minimization approximation [L.-Dai-Ma, 2013]:

$$\begin{array}{ll} \min_{\mathbf{x}} & \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_q^q + \alpha \bar{\mathbf{p}}^T \mathbf{x} \\ \text{s.t.} & \mathbf{0} \leq \mathbf{x} \leq \mathbf{e} \end{array} \tag{8}$$

- Some fundamental questions:
  - Why use the non-convex L_q approximation? Is it better than the convex L_1 approximation? Can the solution of the L_q approximation solve the original sparse problem?
  - Is it easy to solve? Is there any polynomial time algorithm which can solve it to global optimality?
  - Since the problem is non-convex, nonsmooth, and non-Lipschitz, how to solve it efficiently?

L_1 vs L_q: An Illustrative Instance

- Let $\mathbf{A}, \mathbf{b}, \bar{\mathbf{p}}$ in the JPAC problem (3) be

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}, \ \mathbf{b} = 0.5\mathbf{e}, \ \bar{\mathbf{p}} = \mathbf{e}$$

- The optimal solution to problem (3) is

$$\mathbf{x}^* = (0.5, 0.5, 0)^T$$

- For any $\alpha \geq 0$, $\mathbf{x} = \mathbf{0}$ is the unique global minimizer of the L_1 approximation problem.
- For any given $q \in (0, 1)$, if α satisfies

$$0 < \alpha < \bar{\alpha}_q := \min\left\{1 + (0.5)^q, 2^q\right\} - (1.5)^q,$$

then the unique global minimizer of the L_q minimization problem (8) is $\mathbf{x}^*$.

Exact Recovery Result

Theorem (L.-Dai-Ma, 2013)

For any given instance of the JPAC problem (3)*, there exists* $\bar{q} > 0$ *such that when* $q \in (0, \bar{q}]$*, the global solution to its corresponding* L_q *approximation problem is one of the optimal solutions to the JPAC problem* (3)*.*

- This result depends on the special structure of **A** and **b**.
- More works along this direction need to be done...

Complexity Result

Theorem

For any given $0 < q < 1$, *the* L_q *minimization problem* (8) *is NP-hard.*

Reformulation

- For any given $q \in [0, 1]$, problem (8) is equivalent to

$$\begin{aligned} \min_{\mathbf{x},\mathbf{y},\mathbf{z}} \quad & \|\mathbf{y}\|_q^q + \alpha \bar{\mathbf{p}}^T \mathbf{x} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} + \mathbf{y} = \mathbf{b},\ \mathbf{x} + \mathbf{z} = \mathbf{e}, \\ & \mathbf{x} \geq \mathbf{0},\ \mathbf{y} \geq \mathbf{0},\ \mathbf{z} \geq \mathbf{0}. \end{aligned} \tag{9}$$

- Extend the potential reduction algorithm [Ye, 1998; Ge-Jiang-Ye, 2011] to solve problem (9)
  - Potential function:

$$\phi(\mathbf{x},\mathbf{y},\mathbf{z}) = \rho \log\left(\alpha \bar{\mathbf{p}}^T \mathbf{x} + \|\mathbf{y}\|_q^q\right) - \sum_{k=1}^{K} \log\left([\mathbf{x}]_k [\mathbf{y}]_k [\mathbf{z}]_k\right)$$

  - Update rule: the next iterate is chosen as the feasible point that achieves the maximum potential reduction

Polynomial Time Complexity

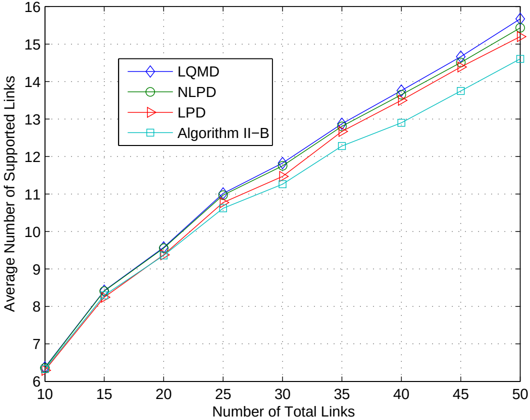
Theorem

The interior-point potential reduction algorithm returns an ϵ-KKT point of problem (9) (equivalent to problem (8)) in no more than $O\left(\left(\frac{K^4}{\epsilon}\right)\log\left(\frac{1}{\epsilon}\right)\right)$ *arithmetic operations.*

Numerical Simulations

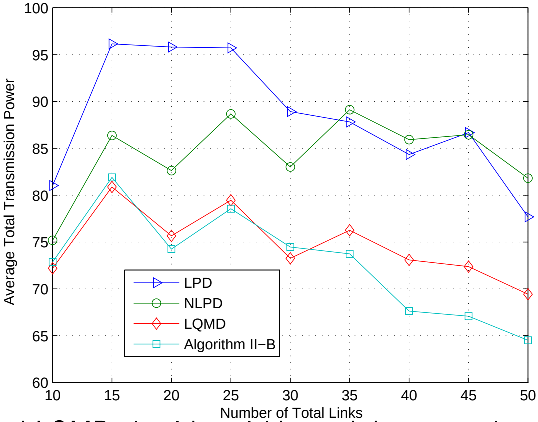
- The same channel parameters are generated as in [Mitliagkas-Sidiropoulos-Swami, 2011]
- Compare the proposed QNMD algorithm with other algorithms including
  - NLPD algorithm [L.-Dai-Luo, 2013]
  - LPD algorithm [Mitliagkas-Sidiropoulos-Swami, 2011]
  - Algorithm II-B [Mahdavi-Doost-Ebrahimi-Khandani, 2010]
- Comparison criteria
  - number of supported links
  - total transmission power
  - executed CPU time
- All figures report the average results for 200 Monte-Carlo runs.

Average Number of Supported Links



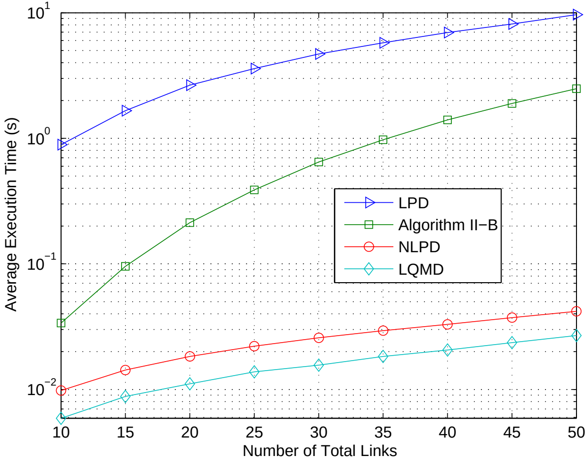
- The proposed LQMD algorithm (with $q = 0.5$) supports (slightly) more links than the NLPD algorithm.
- It is shown in [L.-Dai-Luo, 2013] that the NLPD algorithm can achieve 98% of global optimality in terms of the number of supported links.

Average Total Transmission Power



- The proposed LQMD algorithm yields much better total transmission power performance than the NLPD algorithm.
- The proposed LQMD algorithm exhibits a very good performance in selecting which subset of links to support.

Average CPU Time



Perfect CSI Assumption

- The assumption of the perfect channel state information (CSI) generally does not hold true
  - CSI estimation errors
  - limited CSI feedback
- Even though the instantaneous CSI can be perfectly available, dynamic JPAC in accordance with its variations would lead to excessively high computational and signaling costs.

Channel Distribution Information (CDI)

- The CDI can remain unchanged over a relatively long period of time
- JPAC based on the CDI can therefore be performed on a relatively slow timescale (compared to fast fluctuations of instantaneous channel conditions) $\Longrightarrow$ the overall computational cost and signaling overhead can be significantly reduced
- Chance SINR constrained JPAC formulation
  - maximize the number of *long-term* supported links by using a minimum total transmission power
  - guarantee *short-term* SINR requirements with high probability

Chance SINR Constraints

- Channel gains $\{g_{kj}\}$ are random variables
- Assume the distribution of $\{g_{kj}\}$ are known
- Link k is supported if its SINR outage probability is below a specified tolerance $\epsilon \in (0, 1)$, i.e.,

$$\mathbb{P}\left(\mathrm{SINR}_k(\mathbf{p}) \geq \gamma_k\right) \geq 1 - \epsilon \tag{10}$$

- Computationally intractable

Sample Approximation

- Suppose $\left\{g_{kj}^n\right\}_{n=1}^N$ are N independent samples drawn according to the distribution of $\{g_{kj}\}$
- Approximate the chance SINR constraint (10) by

$$\mathrm{SINR}_k^n(\mathbf{p}) := \frac{g_{kk}^n p_k}{\eta_k + \sum_{j \neq k} g_{kj}^n p_j} \geq \gamma_k, \ \ n = 1, 2, ..., N \tag{11}$$

Approximation Performance

Theorem (Calafiore-Campi, 2005; So-Zhang, 2013)

If the sample size N is greater than

$$\bar{N} := \left\lceil \frac{1}{\epsilon}\left(K - 1 + \ln\frac{1}{\delta} + \sqrt{2(K-1)\ln\frac{1}{\delta} + \ln^2\frac{1}{\delta}}\right)\right\rceil \tag{12}$$

for any $\delta \in (0,1)$, *then the solution to problem*

$$SINR_k^n(\mathbf{p}) \geq \gamma_k,\ k \in \mathcal{K},\ n \in \mathcal{N}$$

will satisfy each of constraint (10) *with probability at least* $1 - \delta$.

Chance SINR Constrained JPAC

- Group sparse formulation [L.-Song-Hong, 2013]:

$$\begin{array}{ll} \min\limits_{\mathbf{x}} & \sum_k \| \max\{\mathbf{c}_k - \mathbf{A}_k \mathbf{x}, \mathbf{0}\} \|_0 + \alpha \bar{\mathbf{p}}^T \mathbf{x} \\ \text{s.t.} & \mathbf{0} \leq \mathbf{x} \leq \mathbf{e} \end{array} \tag{13}$$

- Convex/non-convex approximation-based deflation algorithms

Conclusions

- Joint power and admission control problem
- A computationally efficient algorithm
- Non-convex approximation
- Chance SINR constrained JPAC problem

Thank You!

Email: yafliu@lsec.cc.ac.cn