# INTERNATIONAL WORKSHOP ON CONFORMAL GEOMETRY AND GEOMETRIC PDE JUNE 29 - JULY 3 2015

#### 1. Schedule

Monday June 29 2015 Morning Session Chair: Jie Qing

8:20 - 8: 30 am Opening Ceremony

8:30 - 9:15 am **Speaker:** Alice Sun-Yung Chang, Princeton University. **Title:** Boundary type GJMS operators

9:30 - 10:15 am **Speaker:** Xinan Ma, University of Science and Technology of China **Title:** Gradient Estimates of Mean Curvature Equations and Hessian equation with Neumann Boundary Value Problems

Coffee Break

10:45 - 11:30 am **Speaker:** Lihe Wang, University of Iowa **Title:** Rigidity and regularity of a domain from its capacity

11:45 - 12:30 am **Speaker:** Jeff Streets, UC Irvine **Title:** Generalized Kähler-Ricci flows

Lunch Afternoon Session Chair: Qing Han

2:00 - 2:45 pm **Speaker:** Matt Gursky, University of Notre Dame **Title:** A higher order geometric variational problem from conformal geometry

3:00 - 3:45 pm **Speaker:** Gang Li, BICMR, Peking University **Title:** A gap theorem and curvature estimates for conformally compact Einstein

#### manifolds

Coffee Break 4:15 - 5:00 pm **Speaker:** Pengzi Miao, University of Miami **Title:** On variational properties of static potentials

5:15 - 6:00 pm **Speaker:** Xiping Zhu, Sun Yat-sen University **Title:** Regularity of Harmonic Maps between Singular Spaces

Tuesday June 30 2015 Morning Session Chair: Yuguang Shi

8:30 - 9:15 am **Speaker:** Jiayu Li, University of Science and Technology of China **Title:** On Symplectic Critical Surfaces

9:30 - 10:15 am Speaker: Paul Laurain, Stanford University and Paris 7 Title: Quantization phenomena for conformally invariant problems

Coffee Break

10:45 - 11:30 am Speaker: Jingyi Chen, University of British Columbia Title: Compact branched shrinkers to Lagrangian MCF in the complex plane: Rigidity, compactness, F-stability

11:45 - 12:30 am **Speaker:** Yuxin Ge, University of Paris East Creteil **Title:** Generalized Ginzburg-Landau Equations in high dimensions

Lunch Afternoon Session Chair: Xiaohua Zhu

2:00 - 2:45 pm **Speaker:** Qi Zhang, UC Riverside **Title:** A Li-Yau gradient bound under nearly optimal Ricci curvature condition

3:00 - 3:45 pm **Speaker:** Shiguang Ma, Nankai University **Title:** An ODE approach of constructing CMC surfaces in Riemannian manifolds. Coffee Break

4::15 - 5:00 am **Speaker:** Meijun Zhu, University of Oklahoma **Title:** On curvature equations with negative Sobolev exponent

5:15 - 6:00 pm **Speaker:** Robin Graham, University of Washington **Title:** Higher-dimensional Willmore energies via minimal submanifold asymptotics

Wednesday July 1 2015 Morning Session Chair: Zheng-Chao Han

8:30 - 9:15 am Speaker: Changfeng Gui, University of Connecticut Title: Two-end axially symmetric solutions to the Allen-Cahn equation in entire spaces

9:30 - 10:15 am Speaker: Rod Gover, University of Auckland Title: Conformal hypersurfaces, boundary operators, and related curvatures

Coffee Break

10:45 - 11:30 am **Speaker:** Guozhen Lu, Wayne State University **Title:** Best constants for critical and subcritical Moser-Trudinger inequalities on complete noncompact Riemannian manifolds

11:45 - 12:30 am **Speaker:** Jan Metzger, University of Postdam **Title:** Surfaces minimizing the Willmore energy for small prescribed area

Lunch Conference Intermission

Thursday July 2 2015 Morning Session Chair: Alice Sun-Yung Chang

8:30 - 9:15 am Speaker: Yanyan Li, Rutgers University Title: Compactness of conformal metrics with positive constant Q-curvature 9:30 - 10:15 amSpeaker: Jeffrey Case, Princeton UniversityTitle: Boundary operators associated to the weighted Paneitz operator

Coffee Break

10:45 - 11:30 amSpeaker: Weixiong Chen, Yeshiva UniversityTitle: Direct methods of moving planes, moving spheres, and blowing-ups for the fractional Laplacian

11:45 - 12:30 amSpeaker: Hao Fang, University of IowaTitle: On convergence to footballs and best curvature pinching on conic spheres

Lunch Afternoon Session Chair: Robin Graham

2:00 - 2:45 pm Speaker: Xianzhe Dai, UC Santa Barbara Title: Perelman's entropy functional for manifolds with conical singularity

3:00 - 3:45 pm **Speaker:** Lu Wang, University of Wisconsin **Title:** A topological property for asymptotically conical self-shrinker with small entropy

Coffee Break

4:15 - 5:00 pm Speaker: Fang Wang, Shanghai Jiao Tong University Title: On the scattering operators for Kähler-Einstein manifolds with strictly pseudoconvex CR-infinity

5:15 - 6:00 pm **Speaker:** Dimiter Vasilev, University of New Mexico **Title:** The quaternionic contact Yamabe problem on a 3-Sasakian manifold

Friday July 3 2015 Morning Session Chair: Paul Yang

8:30 - 9:15 am **Speaker:** Xingwang Xu, National University of Singapore Title: Scalar curvature functions on manifolds with vanishing Yamabe invariant

9:30 - 10:15 am Speaker: Zheng-Chao Han, Rutgers University Title: Some remarks on the asymptotic behavior near boundary of conformally flat metrics on domains in  $\mathbb{S}^n$ 

Coffee Break

10:45 - 11:30 am **Speaker:** Jih-Hsin Cheng, Academia Sinica **Title:** The isoperimetric problem in the Heisenberg group

11:15 - 12:30 am **Speaker:** Yuxiang Li, Hsinghua University **Title:** Willmore surfaces embedded in  $R^3$ 

Lunch Afternoon Session Chair: Yanyan Li

2:00 - 2:45 pm **Speaker:** Bo Guan, Ohio State University **Title:** Conformal Hermitian metrics on complex manifolds and related PDEs

3:00 - 3:45 pm **Speaker:** Davi Maximo, Stanford University **Title:** Almost-rigidity of the width of three-manifolds

Coffee Break

4:15 - 5:00 pm **Speaker:** Alessandro Carlotto, Imperial College **Title:**The complexity of a minimal subvariety: analytic data versus topology

5:15 - 6:00 pm **Speaker:** Paul Yang, Princeton University **Title:** CR geometry in 3-D

#### 2. TITLES AND ABSTRACTS

## Speaker: Alessandro Carlotto, Imperial College a.carlotto@imperial.ac.uk

Title: The complexity of a minimal subvariety: analytic data versus topology

Abstract: There are several ways to quantify the "complexity" of a minimal subvariety: on the one hand we have analytic data (like the Morse index, the value of the p-th eigenvalue of the Jacobi operator etc...), on the other we have topological invariants (like the Betti numbers, the sigma invariant etc...). But what is the relation between these pieces of information? I will give a broad overview of some recent results with special emphasis on three of them. First, I will mention a higher-dimensional analogue of the Choi-Schoen compactness theorem which implies equivalence of index and Jacobi eigenvalue bounds. I will then sketch my recent construction in  $S^4$  of minimal hyperspheres of arbitrarily large Morse index and uniformly bounded volume. Lastly, I will move to some other recent work where we show that in various geometric settings (ellipsoids, complex projective spaces, real projective spaces, product of spheres...) the Morse index of a closed minimal hypersurface is bounded from below by an affine function of the first Betti number (with universal coefficients). Some of these results are joint work with Lucas Ambrosio and Benjamin Sharp

Speaker: Jeffrey Case, Princeton University jscase@math.princeton.edu

Title: Boundary operators associated to the weighted Paneitz operator

Abstract: I will describe some conformally covariant operators on a smooth metric measure space with boundary which, when coupled with the weighted Paneitz operator, form an elliptic system. These operators generalize the Robin operator as a boundary operator for the conformal Laplacian. Applications to fractional GJMS operators and sharp Sobolev trace inequalities are presented.

## Speaker: Alice Sun-Yung Chang, Princeton University chang@math.princeton.edu

## Title: Boundary type GJMS operators

Abstract: In around 2001, Graham and Zworski have introduced boundary type GJMS operators through the scattering theory on conformal compact Einstein manifolds. These are boundary operators which generalize the Dirichlet to Neumann operator. In this talk, I will discuss study of these operators via geometry tools (e.g. metric with measures), the positivity property and Sobolev trace inequalities associated with these operators via extension theorems. This is a report of joint works with Jeffrey Case, and separately with Antonio Ache.

### Speaker: Jingyi Chen, University of British Columbia jychen@ubc.ca

Title: Compact branched shrinkers to Lagrangian MCF in the complex plane: Rigidity, compactness, F-stability

Abstract: We will discuss properties of the space of compact self-shrinking solutions to Lagrangian mean curvature flow in the complex plane. For rigidity, we show that there is no compact branched Lagrangian shrinkers of genus zero. For compactness, we show that the space of compact Lagrangian immersed shrinkers can be compactified by the branched ones, under the assumption that the areas and the conformal structures are bounded. We also explain how to prove that compact branched Lagrangian shrinkers are all F-unstable.

### Speaker: Jih-Hsin Cheng, Academia Sinica cheng@math.sinica.edu.tw

### Title: The isoperimetric problem in the Heisenberg group

Abstract: In this talk I will discuss the isoperimetric problem in the Heisenberg group viewed as a subriemannian manifold. First we still have notions of area and mean curvature for a hypersurface in a subriemannian manifold. We study a hypersurface by analyzing its singular set and characteristic curves. In dimension 2 this singular set analysis helps to characterize a C2 smooth closed CMC surface. In higher dimensions we define a notion of umbilicity. We can characterize a C2 smooth closed CMC umbilic hypersurface. The research study in this direction has been conducted by a group of people, besides myself, including Hung-Lin Chiu of Central University, Jenn-Fang Hwang, my institute colleague, Andrea Malchiodi of SNS, Pisa, and Paul Yang of Princeton University.

Speaker: Weixiong Chen, Yeshiva University wchen@yu.edu

Title: Direct methods of moving planes, moving spheres, and blowing-ups for the fractional Laplacian

Abstract: Many conventional approaches on partial differential operators do not work on the nonlocal fractional operator. To overcome this difficulty arising from nonlocalness, Caffarelli and Silvestre introduce the extension method to reduced the problem into a local one in one higher dimensions, which has become a powerful tool in studying such nonlocal problems and has yield a series of fruitful results.

However, due to technical restrictions, sometimes one needs to impose extra conditions when studying the extended problems in higher dimensions, and these conditions may not be necessary if we investigate the original nonlocal problems directly.

In this talk, we will introduce direct methods of moving planes, moving spheres, and blowing-up and re-scaling arguments for the fractional Laplacian. By an elementary approach, we will first show the key ingredients needed in the method of moving planes either in a bounded domain or in the whole space, such as strong maximum principles for anti-symmetric functions, narrow region principles, and decay at infinity. Then, using simple examples, semi-linear equations involving the fractional Laplacian, we will illustrate how this new method of moving planes can be conveniently employed to obtain symmetry and non-existence of positive solutions, under much weaker conditions than in the previous literatures.

We firmly believe that these ideas and approaches can be effectively applied to a wide range of nonlinear problems involving fractional Laplacians or other nonlocal operators.

Speaker: Xianzhe Dai, UC Santa Barbara dai@math.ucsb.edu

Title: Perelman's entropy functional for manifolds with conical singularity

Abstract: Perelman's entropy functional plays an important role in the study of Ricci flow. It is equivalent to the study of an eigenvalue problem for a Schrodinger operator similar to the conformal Laplacian. On a manifold with conical singularity, the potential becomes singular, causing some essential dificulty. The asymptotic behavior of the eigenfunctions is also important for the theory. This is joint work with Changliang Wang.

Speaker: Hao Fang, University of Iowa hao-fang@uiowa.edu

Title: On convergence to footballs and best curvature pinching on conic spheres

Abstract: This is a joint work with Mijia Lai of Shanghai Jiaotong University.

We show that spheres of positive constant curvature with n (n ? 3) conic points converge to a sphere of positive constant curvature with two conic points, or an (American) football in Gromov-Hausdorff topology when the conic angles of the sequence pass from the subcritical case in the sense of Troyanov to the critical case in the limit.

Furthermore, we give the best curvature pinching constant for super-critical conic spheres, generalizing previous results of C.C. Chen-C.S. Lin and D. Bartolucci.

Speaker: Yuxin Ge, University of Paris Est Creteli ge@u-pec.fr

Title: Generalized Ginzburg-Landau Equations in high dimensions

Abstract: In this talk, we present some results on the critical points to the generalized Ginzburg-Landau equations in dimensions  $n \ge 3$  which satisfy a suitable energy bound, but are not necessarily energy-minimizers. When the parameter in the equations tend to zero, such solutions are shown to converge to singular *n*-harmonic maps into spheres which are conformally invariant, and the convergence is strong away from a finite set consisting 1) of the infinite energy singularities of the limiting map, and 2) of points where bubbling off of finite energy *n*-harmonic maps takes place. The latter case is specific to dimensions greater than 2. We also exhibit a criticality condition satisfied by the limiting *n*-harmonic maps which constrains the location of the infinite energy singularities. Finally we construct an example of non-minimizing solutions to the generalized Ginzburg-Landau equations satisfying our assumptions. This is joint work with Etienne Sandier and Peng Zhang.

Speaker: Rod Gover, University of Auckland r.gover@auckland.ac.nz

Title: Conformal hypersurfaces, boundary operators, and related curvatures

Abstract: Among submanifolds, hypersurfaces are particularly important for geometric analysis as they arise as boundaries of domains and manifolds. We discuss the treatment of conformal hypersurfaces, and in particular the construction of canonical invariant differential operators along a hypersurface in a conformal manifold. The operators constructed have applications in the construction of higher order conformally invariant Dirichlet-to-Neumann operators (that may be interpreted as conformally invariant fraction powers of the Laplacian). Associated to the operators are interesting curvature quantities that in a conformal sense generalise the mean curvature and the T-curvature of Chang-Qing. The operators also shed light on conjectures of Juhl. This is joint work with Larry Peterson.

Speaker: Robin Graham, University of Washington robin@math.washingon.edu

Title: Higher-dimensional Willmore energies via minimal submanifold asymptotics

Abstract: This talk will describe a derivation of a conformally invariant energy for an even-dimensional submanifold of a Riemannian manifold generalizing the Willmore energy of a surface. The energy and its associated Euler-Lagrange equation both arise naturally upon considering the asymptotics of minimal submanifolds in asymptotically Poincaré-Einstein spaces associated to the background conformal manifold.

Speaker: Bo Guan, Ohio State University guan@math.ohio-state.edu

Title: Conformal Hermitian metrics on complex manifolds and related PDEs

Abstract: We are concerned with the problem of finding conformal metrics with certain special properties on Hermitian manifolds, and study the nonlinear elliptic equations arising form the problem.

## Speaker: Changfeng Gui, University of Connecticut changfeng.gui@uconn.edu

Title: Two-end axially symmetric solutions to the Allen-Cahn equation in entire spaces

Abstract: In this talk, I will present some results on axially symmetric solutions to the Allen-Cahn equation in entire spaces. In particular, a complete branch of axially symmetric entire solutions to the Allen-Cahn equation in  $\mathbb{R}^3$  will be constructed. The nodal sets of these solutions behave asymptotically like catenoids at the two ends of the axis. These solutions are monotone in the radial direction and even in the direction of the axis after possible translation, and have finite Morse indices. The compactness of solutions and the linearized equations are carefully investigated and play important roles in the analysis. This talk is based on joint works with Yong Liu and Juncheng Wei.

Speaker: Matthew Gursky , University of Notre Dame Matthew.J.Gursky.1@nd.edu

Title: A higher order geometric variational problem from conformal geometry

Abstract: In this talk I will describe a fourth order variational problem with critical nonlinearity. I will begin with an overview, explaining its connection to sharp Sobolev inequalities and its geometric interpretation. I will then describe some recent work which shows that a maximum principle holds, despite being higher order, and present some existence results. This is joint work with A. Malchiodi (SNS), Y. Lin (Michigan), and F. Han (Courant).

# Speaker: Zheng-Chao Han, Rutgers University zchan@math.rutgers.edu

Title: Some remarks on the asymptotic behavior near boundary of conformally flat metrics on domains in  $\mathbb{S}^n$ 

Abstract: Complete conformally flat metrics often arise in conformal geometry, so there is need to understand both geometric and analytic constructions and properties of such metrics. In this talk, I will provide a review of some earlier results, and add some more recent observations.

# Speaker: Paul Laurain, Stanford University and Paris 7 plaurain@stanford.edu

Title: Quantization phenomena for conformally invariant problems

Abstract: This talk is devoted to a series of papers we have published with T. Rivire. First, we have been interested in giving a unified proof of some "classical" quantization phenomena for problems such as harmonic maps, J-holomorphic curves or prescribed mean curvature. Then, since this proof relies only on the common dominator of theses problems, namely conformal invariance, we have been able to applied this theory to solve open questions in conformal geometry. I will notably explain how this theory permits to study the moduli space of Willmore surfaces, first with fixed conformal class, and if I have enough time I will finally talk about how to deal with the case of degenerating conformal classes.

Speaker: Gang Li, BICMR, Peking University renxing3@163.com

Title: A gap theorem and curvature estimates for conformally compact Einstein manifolds

Abstract: For a fourth dimensional Conformally Compact Einstein manifold  $(M^4, g)$ , the renormalized volume is a global conformal invariant. Based on assumption on the renormalized volume, we obtain a gap theorem and curvature estimates on (M, g). We also obtain a curvature pinching theorem on Conformally Compact Einstein manifold  $(M^n, g)$  with  $n \ge 4$ , provided that the Yamabe constant of the conformal infinity is large. This is a joint work with Professor Jie Qing and Professor Yuguang Shi.

Speaker: Jiayu Li, University of Science and Technology of China jiayuli1963@gmail.com

Title: On Symplectic Critical Surfaces

Abstract: In this talk, we introduce new functionals to study the existence of holomorphic curves in Kähler surfaces. We study the properties of the critical surfaces of the functionals.

Speaker: Yanyan Li, Rutgers University yyli@math.rutgers.edu

Title: Compactness of conformal metrics with positive constant Q-curvature

Abstract: Let (M, g) be an  $n \ge 5$  dimensional smooth compact Riemannian manifold of positive Yamabe type, which is not conformally equivalent to the standard sphere. We prove compactness of conformal metrics of g with positive constant Q-curvature provided that (M, g) is locally conformally flat, or  $5 \le n \le 9$ . We also prove the compactness result in dimension  $n \ge 8$ , provided that the Weyl tensor of g does not vanish anywhere. This is a joint work with Jingang Xiong.

**Speaker**: Yuxiang Li, Hsinghua University yxli@math.tsinghua.edu.cn

Title: Willmore surfaces embedded in  $\mathbb{R}^3$ 

Abstract: We will prove that any Willmore embedding of  $\mathbb{C}$  into  $R^3$  is a plane. Then we will show that that if  $\Sigma_k$  is a sequence of closed Willmore surface embedded in  $R^3$  with  $W(\Sigma_k) < C$ , and if the conformal class of  $\Sigma_k$  converges in the moduli space, then we can find a Möbius transformation  $\sigma_k$ , such that a subsequence of  $\sigma_k(\Sigma_k)$  converges smoothly.

## Speaker: Guozhen Lu, Wayne State University gzlu@wayne.edu

Title: Best constants for critical and subcritical Moser-Trudinger inequalities on complete noncompact Riemannian manifolds

Abstract: In this talk we will report some recent joint work with Jungang Li on sharp critical and subcritical Moser-Trudinger inequalities on complete noncompact Riemannian manifolds under some curvature conditions. Our philosophy is to use an approach from sharp local Moser-Trudinger inequalities on compact domains to deduce sharp global inequalities on complete noncompact Riemannian manifolds using the level sets of functions under consideration and thus avoid the argument of symmetrization which is not available on Riemannian manifolds.

Speaker: Shiguang Ma, Nankai University msgdyx8741@163.com

Title: An ODE approach of constructing CMC surfaces in Riemannian manifolds

Abstract: I will talk about an ODE approach of constructing unstable CMC spheres in Riemannian manifolds which are close to Euclidean space. These CMC surfaces also serve as examples of singular limits of a sequence of Delaunay type CMC surfaces in Riemannian manifolds.

Speaker: Xinan Ma, University of Science and Technology of China xinan@ustc.edu.cn

Title: Gradient Estimates of Mean Curvature Equations and Hessian equation with Neumann Boundary Value Problems

Abstract: In this paper, we use the maximum principle to get the gradient estimate for the solutions of the prescribed mean curvature equation with Neumann boundary value problem, which gives a positive answer for the question raised by Lieberman. As a consequence, we obtain the corresponding existence theorem for a class of mean curvature equations. We also study the Hessian equation with Neumann boundary value problem. This is a joint works with Jinju Xu and Guohuan Qiu.

Speaker: Davi Maximo, Stanford University maximo@stanford.edu

Title: Almost-rigidity of the width of three-manifolds

Abstract: We show that a positively curved three-manifold with almost maximal width is nearly round.

# Speaker: Jan Metzger, University of Potsdam jan.metzger@uni-potsdam.de

Title: Surfaces minimizing the Willmore energy for small prescribed area

Abstract: In this talk I will consider the Willmore functional for surfaces in three dimensional Riemannian manifolds subject to (small) prescribed area. The goal is to discuss old and new results on the existence of such surfaces and their interaction with the ambient geometry. This is joint work with Tobias Lamm and Felix Schulze.

# Speaker: Pengzi Miao, University of Miami p.miao@math.miami.edu

Title: On variational properties of static potentials

Abstract: We discuss solutions to the static, vacuum Einstein equation on asymptotically flat manifolds without a sign assumption imposed on the static potential. By analyzing the conformally related Fermat metric, we show that any noncompact component of the zero set of a static potential must be area minimizing. As applications, we obtain rigidity results for manifolds admitting unbounded static potentials.

Speaker: Jeff Streets, UC Irvine jstreets@math.uci.edu

Title: Generalized Kähler-Ricci flows

Abstract: I will discuss a geometric evolution equation which extends Kahler Ricci flow into the setting of generalized Kähler geometry. I will discuss the reduction of this flow to a parabolic fully nonlinear nonconvex PDE. I will show a priori estimates for this flow which lead to global existence results and explain the geometric and topological implications of these results. This is joint work with G. Tian.

Speaker: Dimiter Nickolov Vassilev, University of New Mexico

Title: The quaternionic contact Yamabe problem on a 3-Sasakian manifold

Abstract: We will explain the solution of the quaternionic contact (qc) Yamabe problem on the 3-Sasakian round sphere. Uniqueness in a qc conformal class containing a 3-Sasakain metric will be shown by exploiting a relation between infinitesimal qc automorphisms and a Lichnerowicz-Obata type result for the first eigenvalue of the sublaplacian. A unified approach to the Riemannian, CR and qc cases of the considered problem will be stressed. Speaker: Fang Wang, Shanghai Jiao Tong University fangwang1984@sjtu.edu.cn

Title: On the scattering operators for Kähler-Einstein manifolds with strictly pseudoconvex CR-infinity

Abstract: In this talk, I will talk about the positivity of scattering operators for Kähler-Einstein manifolds with strictly pseudoconvex CR-infinity which has positive Webster scalar curvature. The result is parallel to Guillarmou-Qing's positivity result for scattering operators for Poincaré-Einstein manifolds. I will also give an energy identity between the boundary and the interior, which is parallel to Case-Changs result for Poincaré-Einstein manifolds.

Speaker: Lihe Wang, University of Iowa lihe-wang@uiowa.edu

Title: Rigidity and regularity of a domain from its capacity

Abstract: We will prove a rigidity theorem using a capacity condition on a domain of a set, ie, we will show that the set is the half space if it is like a cone in term of capacity near each point near the boundary of the domain. The consequences to the regularity of a domain will be also discussed.

Speaker: Lu Wang, University of Wisconsin Madison lwang@math.jhu.edu

Title: A topological property for asymptotically conical self-shrinker with small entropy

Abstract: For any asymptotically conical self-shrinker with entropy less than or equal to that of a cylinder we show that the link of the asymptotic cone must separate the unit sphere into exactly two connected components, both diffeomorphic to the self-shrinker. Combining this with recent work of Brendle, we conclude that the round sphere uniquely minimizes the entropy among all non-flat two-dimensional self-shrinkers. This is joint work with Jacob Bernstein.

Speaker: Xingwang Xu, National University of Singapore matxuxw@nus.edu.sg

Title: Scalar curvature functions on manifolds with vanishing Y amabe invariant

Abstract: In this talk, we describe an alternative approach to some quantitative results of Escobar and Schoen for metrics of prescribed scalar curvature on closed Riemannian manifolds with vanishing Yamabe invariant via a negative gradient curvature flow. The main result is to remove the so-called flatness condition proposed by Escobar and Schoen; hence answering a conjecture by Kadzan and Warner affirmatively. This talk is based on my joint work with Ngo Quoc Anh Speaker: Paul Yang, Princeton University yang@math.princeton.edu

Title: CR geometry in 3-D

Abstract: I will talk about several Sobolev inequalities for CR geometry in 3-D. Each one is related with conformally covariant operators and its associated curvature. The most familiar one is the analogue of the Yamabe equation, and the most recent one has to do with the newly introduced Q-prime curvature.

# Speaker: Qi Zhang, UC Riverside

Title: A Li-Yau gradient bound under nearly optimal Ricci curvature condition

Abstract: We prove Li-Yau type gradient bound for the heat equation either on fixed manifolds or on Ricci flows. In the former case the curvature condition is  $|Ric^-| \in L^p$  for some p > n/2, or  $\sup_M \int_M |Ric^-|^2(y)d^{2-n}(x,y)dy < \infty$ , where n is the dimension of the manifold. In the later case, then one only needs scalar curvature being bounded. We will explain why the conditions are nearly optimal and give an application on extending Colding-Naber's result. The Li-Yau bound on the heat equation seems to be the first one allowing Ricci curvatures not bounded from below.

This is joint work with Richard H. Bamler and Meng Zhu.

Speaker: Meijun Zhu, University of Oklahoma mzhu@ou.edu

Title: On curvature equations with negative Sobolev exponent

Abstract: In this talk, I shall describe our recent work on curvature equations with negative exponent (prescribing curvature on  $S^n$  or Yamabe type problem on general compact manifolds). The basic equation is

$$(-\Delta)^{\frac{\alpha}{2}}u = Ru^{\frac{n+\alpha}{n-\alpha}}, \quad u > 0, \quad \text{in } R^n,$$

where  $\alpha > n$ . The main difference between  $\alpha > n$  and  $\alpha < n$  will be addressed.

We obtain the existence results based on blowup analysis, a new energy criterion, and a newly established reversed sharp Hardy-Littlewood-Sobolev inequality.

Speaker: Xiping Zhu, Sun Yat-sen University stszxp@mail.sysu.edu.cn

Title: Regularity of Harmonic Maps between Singular Spaces

Abstract: M. Gromov and R. Schoen in 1992 initiated to study the theory of harmonic maps into singular spaces, motivated by the p-adic superrigidity for lattices in group of rank one. In 1997, J. Jost and F. H. Lin, independently proved that every energy minimizing harmonic map from an Alexandrov space with curvature bounded from below to an Alexandrov space with non-positive curvature is locally Hölder continuous. Meanwhile, F. H. Lin proposed an open question: can the Hölder continuity be improved to Lipschitz continuity? J. Jost also asked a similar problem about Lipschitz regularity of harmonic maps between singular spaces. In this talk I will present an affirmative answer to it. This is a joint work with Hui-Chun Zhang.

#### 3. Participants

Antonio Ache, Princeton University Vincent Bonini, Cal Poly San Luis Obispo Alessandro Carlotto, Imperial College Jeffrey Case, Princeton University Alice Sun-Yung Chang, Princeton University Jingyi Chen, University of British Columbia Jih-Hsin Cheng, Academia Sinica Weixiong Chen, Yeshiva University Xianzhe Dai, UC Santa Barbara Qi Ding, Shanghai Center for Mathematical Sciences, Fudan University Hao Fang, University of Iowa Yuxin Ge, University of Paris Est Creteli Mara del Mar Gonzlez Nogueras, Universitat Politecnica de Catalunya Rod Gover, University of Auckland Robin Graham, University of Washington Bo Guan, Ohio State University Changfeng Gui, University of Connecticut Matthew Gursky, University of Notre Dame Zhengchao Han, Rutgers University Fengbo Hang, New York University Seongtag Kim, Inha University Paul Laurain, Stanford University and Paris 7 Chao Li, Stanford University Jiavu Li, University of Science and Technology of China Yanyan Li, Rutgers University Yuxiang Li, Hsinghua University Longzhi Lin, UC Santa Cruz Guozhen Lu, Wavne State University Xinan Ma, University of Science and Technology of China Davi Maximo, Stanford University Jan Metzger, University of Potsdam Pengzi Miao, University of Miami Nick Reichert, University of Washington Weimin Sheng, Zhejiang University Jeff Streets, UC Irvine Dimiter Nickolov Vassilev, University of New Mexico Fang Wang, Shanghai Jiao Tong University Guofang Wang, University of Freiburg

Lihe Wang, University of Iowa Lu Wang, University of Wisconsin Madison Juncheng Wei, University of British Columbia Xingwang Xu, National University of Singapore Paul Yang, Princeton University Qi Zhang, UC Riverside Yu Zheng, East China Normal University Meijun Zhu, University of Oklahoma Xiaohua Zhu, Peking University Xiping Zhu, Sun Yat-sen University

### **Conference Secretary:**

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