Douglas-Rachford method, ADMM and PDHG

Acknowledgement: this slides is based on Prof. Lieven Vandenberghes lecture notes
Outline

1. Douglas-Rachford splitting method
2. examples
3. alternating direction method of multipliers
4. image deblurring example
5. convergence
Douglas-Rachford splitting algorithm

Consider

\[
\min \ f(x) = g(x) + h(x)
\]

\(g\) and \(h\) are closed convex functions

**Douglas-Rachford iteration:** starting at any \(z^{(0)}\), repeat

\[
\begin{align*}
x^{(k)} &= \text{prox}_{th}(z^{(k-1)}) \\
y^{(k)} &= \text{prox}_{tg}(2x^{(k)} - z^{(k-1)}) \\
z^{(k)} &= z^{(k-1)} + y^{(k)} - x^{(k)}
\end{align*}
\]

- \(t\) is a positive constant (simply scales the objective)
- useful when \(g\) and \(h\) have inexpensive prox-operators
- under weak conditions (existence of a minimizer), \(x^{(k)}\) converges
Equivalent form

- start iteration at $y$-update
  \[
  y^+ = \text{prox}_{tg}(2x - z); \quad z^+ = z + y^+ - x; \quad x^+ = \text{prox}_{th}(z^+)
  \]
- switch $z$- and $x$-updates
  \[
  y^+ = \text{prox}_{tg}(2x - z); \quad x^+ = \text{prox}_{th}(z + y^+ - x); \quad z^+ = z + y^+ - x
  \]
- make change of variables $w = z - x$

alternate form of DR iteration: start at $x^{(0)} \in \text{dom } h, w^{(0)} \in t\partial h(x^{(0)})$

  \[
  y^+ = \text{prox}_{tg}(x - w) \\
  x^+ = \text{prox}_{th}(y^+ + w) \\
  w^+ = w + y^+ - x^+
  \]
Interpretation as fixed-point iteration

Douglas-Rachford iteration can be written as

\[ z^{(k)} = F(z^{(k-1)}) \]

where \( F(z) = z + \text{prox}_{tg}(2\text{prox}_{th}(z) - z) - \text{prox}_{th}(z) \)

fixed points of \( F \) and minimizers of \( g + h \)

- if \( z \) is a fixed point, then \( x = \text{prox}_{th}(z) \) is a minimizer:

  \[ z = F(z), \quad x = \text{prox}_{th}(z) \Rightarrow \text{prox}_{tg}(2x - z) = x = \text{prox}_{th}(z) \]
  \[ \Rightarrow x - z \in t\partial g(x); \quad z - x \in t\partial h(x) \]
  \[ \Rightarrow 0 \in t\partial g(x) + t\partial h(x) \]

- if \( x \) is a minimizer and \( u \in t\partial g(x) \cap -t\partial h(x) \), then \( x - u = F(x - u) \)
Douglas-Rachford iteration with relaxation

fixed-point iteration with relaxation

\[ z^+ = z + \rho (F(z) - z) \]

1 \(<\rho \<2\) is overrelaxation, 0 \(<\rho \<1\) is underrelaxation

first version of DR method

\[ x^+ = \text{prox}_{th}(z) \]
\[ y^+ = \text{prox}_{tg}(2x^+ - z) \]
\[ z^+ = z + \rho (y^+ - x^+) \]

alternate version

\[ y^+ = \text{prox}_{tg}(x - w) \]
\[ x^+ = \text{prox}_{th}((1 - \rho)x + \rho y^+ + w) \]
\[ w^+ = w + \rho y^+ + (1 - \rho)x - x^+ \]
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2. Examples
3. Alternating direction method of multipliers
4. Image deblurring example
5. Convergence
Sparse inverse covariance selection

\[ \min \ tr(CX) - \log \det X + \rho \sum_{i > j} |X_{ij}| \]

variable is \( X \in S^n \); parameters \( C \in S^n_{++} \) and \( \rho > 0 \) are given

**Douglas-Rachford splitting**

\[ g(X) = tr(CX) - \log \det X, \quad h(x) = \rho \sum_{i > j} |X_{ij}| \]

- \( X = \text{prox}_{tg}(\hat{X}) \) is positive solution of \( C - X^{-1} + (1/t)(X - \hat{X}) = 0 \) easily solved via eigenvalue decomposition of \( \hat{X} - tC \)

- \( X = \text{prox}_{th}(\hat{X}) \) is soft-thresholding
Spingarn’s method of partial inverses

equality constrained convex problem

\[
\begin{align*}
\min & \quad h(x) \\
\text{s.t.} & \quad x \in V
\end{align*}
\]

\( h \) a closed convex function; \( V \) a subspace

Douglas-Rachford splitting: take \( g = I_V \) (indicator of \( V \) )

\[
\begin{align*}
x^+ &= \text{prox}_{th}(z) \\
y^+ &= P_V(2x^+ - z) \\
z^+ &= z + y^+ - x^+
\end{align*}
\]
Application to composite optimization problem

\[ \min f_1(x) + f_2(Ax) \]

\(f_1\) and \(f_2\) have simple prox-operators

- equivalent to minimizing \(h(x_1, x_2)\) over subspace \(V\) where

  \[ h(x_1, x_2) = f_1(x_1) + f_2(x_2), \quad V = \{(x_1, x_2) \mid x_2 = Ax_1\} \]

- prox\(_th\) is separable: \(\text{prox}_{th}(x_1, x_2) = (\text{prox}_{tf_1}(x_1), \text{prox}_{tf_2}(x_2))\)

- projection of \((x_1, x_2)\) on \(V\) reduces to linear equation:

  \[ P_V(x_1, x_2) = \begin{pmatrix} I \\ A \end{pmatrix} (I + A^T A)^{-1} (x_1 + A^T x_2) \]

  \[ = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} A^T \\ -I \end{pmatrix} (I + A^T A)^{-1} (x_2 - Ax_1) \]
Decomposition of separable problems

\[
\min \sum_{j=1}^{n} f_j(x_j) + \sum_{i=1}^{m} g_i(A_{i1}x_1 + \cdots + A_{in}x_n)
\]

- same problem as the lecture on "dual proximal gradient method", but without strong convexity assumption
- we assume the functions \(f_j\) and \(g_i\) have inexpensive prox-operators

**equivalent formulation**

\[
\min \sum_{j=1}^{n} f_j(x_j) + \sum_{i=1}^{m} g_i(y_{i1} + \cdots + y_{in})
\]

s.t. \(y_{ij} = A_{ij}x_j, \ i = 1, \ldots, m; \ j = 1, \ldots, n\)

- prox-operator of cost involves uncoupled prox-evaluations for \(f_j, g_i\)
- projection on constraint set reduces to \(n\) independent linear equations
Decomposition of separable problems

**second equivalent formulation** with extra splitting variables $x_{ij}$:

$$\min \sum_{j=1}^{n} f_j(x_j) + \sum_{i=1}^{m} g_i(y_{i1} + \cdots + y_{in})$$

s.t.  
$$x_{ij} = x_j, \quad i = 1, \ldots, m; \quad j = 1, \ldots, n$$  
$$y_{ij} = A_{ij}x_{ij}, \quad i = 1, \ldots, m; \quad j = 1, \ldots, n$$

- make first set of constraints part of domain of $f_j$:

$$\tilde{f}_j(x_j, x_{1j}, \cdots, x_{mj}) = \begin{cases} f_j(x_j) & x_{ij} = x_j, \quad i = 1, \ldots, m \\ +\infty & \text{otherwise} \end{cases}$$

prox-operator of $\tilde{f}_j$ reduces to prox-operator of $f_j$

- projection on other constraints involves $mn$ independent linear equations
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Dual application of Douglas-Rachford method

separable convex problem

\[
\begin{align*}
\min & \quad f_1(x_1) + f_2(x_2) \\
\text{s.t.} & \quad A_1 x_1 + A_2 x_2 = b
\end{align*}
\]

dual problem

\[
\begin{align*}
\max & \quad -b^T z - f_1^*(-A_1^T z) - f_2^*(-A_2^T z)
\end{align*}
\]

we apply the Douglas-Rachford method (page 3) to minimize

\[
\begin{align*}
g(z) & = b^T z + f_1^*(-A_1^T z) \\
h(z) & = f_2^*(-A_2^T z)
\end{align*}
\]
Douglas Rachford on the dual

\[ y^+ = \text{prox}_{tg}(z - w), \quad z^+ = \text{prox}_{th}(y^+ + w), \quad w^+ = w + y^+ - z^+ \]

first line: use result in "lect-dualProxGrad.pdf" to compute
\[ y^+ = \text{prox}_{tg}(z - w) \]

\[ \hat{x}_1 = \arg\min_{x_1} \left( f_1(x_1) + z^T (A_1 x_1 - b) + \frac{t}{2} \| A_1 x_1 - b - w/t \|_2^2 \right) \]
\[ y^+ = z - w + t(A_1 \hat{x}_1 - b) \]

second line: similarly, compute \( z^+ = \text{prox}_{th}(z + t(A_1 \hat{x}_1 - b)) \)

\[ \hat{x}_2 = \arg\min_{x_1} \left( f_1(x_2) + z^T A_2 x_2 + \frac{t}{2} \| A_1 \hat{x}_1 + A_2 x_2 - b \|_2^2 \right) \]
\[ z^+ = z + t(A_1 \hat{x}_1 + A_2 \hat{x}_2 - b) \]

third line reduces to \( w^+ = -tA_2 \hat{x}_2 \)
Alternating direction method of multipliers

1. minimize augmented Lagrangian over $x_1$

$$x_1^{(k)} = \arg\min_{x_1} \left( f_1(x_1) + (z^{(k-1)})^T A_1 x_1 + \frac{t}{2} \| A_1 x_1 + A_2 x_2^{(k-1)} - b \|_2^2 \right)$$

2. minimize augmented Lagrangian over $x_2$

$$x_2^{(k)} = \arg\min_{x_2} \left( f_2(x_2) + (z^{(k-1)})^T A_2 x_2 + \frac{t}{2} \| A_1 x_1^{(k)} + A_2 x_2 - b \|_2^2 \right)$$

3. dual update

$$z^{(k)} = z^{(k-1)} + t(A_1 x_1^{(k)} + A_2 x_2^{(k)} - b)$$

also known as split Bregman method
Comparison with other multiplier methods

**alternating minimization method** with $g(y) = I_{\{b\}}(y)$

- same dual update, same update for $x_2$
- $x_1$-update in alternating minimization method is simpler:

$$x_1^{(k)} = \arg\min_{x_1} \left( f_1(x_1) + (z^{(k-1)})^T A_1 x_1 \right)$$

- ADMM does not require strong convexity of $f_1$

**augmented Lagrangian method** with $g(y) = I_{\{b\}}(y)$

- dual update is the same
- AL method requires joint minimization of the augmented Lagrangian

$$\min_{x_1, x_2} f_1(x_1) + f_2(x_2) + (z^{(k-1)})^T (A_1 x_1 + A_2 x_2) + \frac{t}{2} \|A_1 x_1 + A_2 x_2 - b\|_2^2$$
Application to composite optimization (method 1)

\[
\min f_1(x) + f_2(Ax)
\]

apply ADMM to

\[
\min f_1(x_1) + f_2(x_2)
\]

s.t. \( Ax_1 = x_2 \)

- augmented Lagrangian is

\[
f_1(x_1) + f_2(x_2) + \frac{t}{2} \| Ax_1 - x_2 + z/t \|_2^2
\]

- \( x_1 \)-update requires minimization of \( f_1(x_1) + \frac{t}{2} \| Ax_1 - x_2 + z/t \|_2^2 \)

- \( x_2 \)-update is evaluation of \( \text{prox}_{t^{-1}f_2} \)
Application to composite optimization (method 2)

introduce extra ‘splitting’ or ‘dummy’ variable $x_3$

$$
\begin{align*}
\min & \quad f_1(x_3) + f_2(x_2) \\
\text{s.t.} & \quad \begin{pmatrix} A \\ I \end{pmatrix} x_1 = \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}
\end{align*}
$$

- alternate minimization of augmented Lagrangian over $x_1$ and $(x_2, x_3)$

$$
f_1(x_3) + f_2(x_2) + \frac{t}{2} \left( \|A x_1 - x_2 + z_1/k\|_2^2 + \|x_1 - x_3 + z_2/k\|_2^2 \right)
$$

- $x_1$-update: linear equation with coefficient $I + A^T A$
- $(x_2, x_3)$-update: decoupled evaluations of $\text{prox}_{t^{-1}f_1}$ and $\text{prox}_{t^{-1}f_2}$
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Image blurring model

\[ b = Kx_t + w \]

- \( x_t \) is unknown image
- \( b \) is observed (blurred and noisy) image; \( w \) is noise
- \( N \times N \)-images are stored in column-major order as vectors of length \( N^2 \)

**blurring matrix** \( K \)
- represents 2D convolution with space-invariant point spread function
- with periodic boundary conditions, block-circulant with circulant blocks
- can be diagonalized by multiplication with unitary 2D DFT matrix \( W \):
  \[ K = W^H \text{diag}(\lambda) W \]

Equations with coefficient \( I + K^T K \) can be solved in \( O(N^2 \log N) \) time
Total variation deblurring with 1-norm

\[ \min \| Kx - b \|_1 + \gamma \| Dx \|_{tv} \]

s.t. \( 0 \leq x \leq 1 \)

second term in objective is **total variation penalty**

- \( Dx \) is discretized first derivative in vertical and horizontal direction:

\[
\begin{pmatrix}
-1 & 0 & 0 & \cdots & 0 & 0 & 1 \\
1 & -1 & 0 & \cdots & 0 & 0 & 0 \\
0 & 1 & -1 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & -1 & 0 & 0 \\
0 & 0 & 0 & \cdots & 1 & -1 & 0 \\
0 & 0 & 0 & \cdots & 0 & 1 & -1 \\
\end{pmatrix}
\]

- \( \| \cdot \|_{tv} \) is a sum of Euclidean norms:

\[
\| (u, v) \|_{tv} = \sum_{i=1}^{n} \sqrt{u_i^2 + v_i^2}
\]
an example of a composite optimization problem

\[
\min \quad f_1(x) + f_2(Ax)
\]

with \( f_1 \) the indicator of \([0, 1]^n\) and \( A = \begin{pmatrix} K \\ D \end{pmatrix} \), \( f_2(u, v) = \|u\|_1 + \gamma \|v\|_{tv} \)

\[
\min \quad \|u\|_1 + \gamma \|v\|_{tv}, \quad \text{s.t.} \quad u = Kx - b, \quad v = Dx, \quad y = x, \quad 0 \leq y \leq 1
\]

**primal DR method** and **ADMM** require:

- decoupled prox-evaluations of \( \|u\|_1 \) and \( \|v\|_{tv} \), and projections on \( C \)
- solution of linear equations with coefficient matrix

\[
I + K^T K + D^T D
\]

solvable in \( O(N^2 \log N) \) time
Example

- 1024 × 1024 image, periodic boundary conditions
- Gaussian blur
- salt-and-pepper noise (50% pixels randomly changed to 0/1)
Convergence

The cost per iteration is dominated by 2D FFTs.
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Nonexpansiveness

if $u = \text{prox}_h(x)$, $v = \text{prox}_h(y)$, then

$$(u - v)^\top (x - y) \geq \|u - v\|_2$$

prox$_h$ is \textit{firmly nonexpansive}, or \textit{co-coercive} with constant 1

- follows from characterization of proximal mapping and monotonicity

$$x - u \in \partial h(u), y - v \in \partial h(v) \quad \Rightarrow \quad (x - u - y + v)^\top (u - v) \geq 0$$

- implies (from Cauchy-Schwarz inequality)

$$\|\text{prox}_h(x) - \text{prox}_h(y)\|_2 \leq \|x - y\|_2$$

prox$_h$ is \textit{nonexpansive}, or \textit{Lipschitz continuous} with constant 1
Douglas-Rachford iteration mappings

Define iteration map $F$ and negative step $G$

$$F(z) = z + \text{prox}_{tg}(2\text{prox}_{th}(z) - z) - \text{prox}_{th}(z)$$

$$G(z) = z - F(z)$$

$$= \text{prox}_{th}(z) - \text{prox}_{tg}(2\text{prox}_{th}(z) - z)$$

• $F$ is firmly nonexpansive (co-coercive with parameter 1)

$$(F(z) - F(\hat{z}))^T(z - \hat{z}) \geq \|F(z) - F(\hat{z})\|_2^2 \quad \forall z, \hat{z}$$

• Implies that $G$ is firmly nonexpansive:

$$(G(z) - G(\hat{z}))^T(z - \hat{z})$$

$$= \|G(z) - G(\hat{z})\|_2^2 + (F(z) - F(\hat{z}))^T(z - \hat{z}) - \|F(z) - F(\hat{z})\|_2^2$$

$$\geq \|G(z) - G(\hat{z})\|_2^2$$
Proof.

firm nonexpansiveness of $F$

- define $x = \text{prox}_{th}(z)$, $\hat{x} = \text{prox}_{th}(\hat{z})$, and
  \[ y = \text{prox}_{tg}(2x - z), \quad \hat{y} = \text{prox}_{tg}(2\hat{x} - \hat{z}) \]

- substitute expressions $F(z) = z + y - x$ and $F(\hat{z}) = \hat{z} + \hat{y} - \hat{x}$:

  \[
  (F(z) - F(\hat{z}))^T (z - \hat{z}) \\
  \geq (z + y - x - \hat{z} - \hat{y} + \hat{x})^T (z - \hat{z}) - (x - \hat{z})^T (z - \hat{z}) + \|x - \hat{x}\|_2^2 \\
  = (y - \hat{y})^T (z - \hat{z}) + \|z - x - \hat{z} + \hat{x}\|_2^2 \\
  = (y - \hat{y})^T (2x - z - 2\hat{x} + \hat{z}) - \|y - \hat{y}\|_2^2 + \|F(z) - F(\hat{z})\|_2^2 \\
  \geq \|F(z) - F(\hat{z})\|_2^2
  \]

inequalities use firm nonexpansiveness of $\text{prox}_{th}$ and $\text{prox}_{tg}$

- $x - \hat{x})^T (z - \hat{z}) \geq \|x - \hat{x}\|_2^2$,  
  \( 2x - z - 2\hat{x} + \hat{z})^T (y - \hat{y}) \geq \|y - \hat{y}\|_2^2 \)
Convergence result

\[ z^{(k)} = (1 - \rho_k)z^{(k-1)} + \rho_k F(z^{(k-1)}) \]
\[ = z^{(k-1)} - \rho_k G(z^{(k-1)}) \]

**assumptions**

- optimal value \( f^* = \inf_x (g(x) + h(x)) \) is finite and attained
- \( \rho_k \in [\rho_{\min}, \rho_{\max}] \) with \( 0 < \rho_{\min} < \rho_{\max} < 2 \)

**result**

- \( z^{(k)} \) converges to a fixed point \( z^* \) of \( F \)
- \( x^{(k)} = \text{prox}_{th}(z^{(k-1)}) \) converges to a minimizer \( x^* = \text{prox}_{th}(z^*) \) (follows from continuity of \( \text{prox}_{th} \))
Proof.

Let $z^*$ be any fixed point of $F(z)$ (zero of $G(z)$). Consider iteration $k$ (with $z = z^{(k-1)}$, $\rho = \rho_k$, $z^+ = z^{(k)}$):

$$
\|z^+ - z^*\|_2^2 - \|z - z^*\|_2^2 = 2(z^+ - z)^T(z - z^*) + \|z^+ - z\|_2^2 \\
= -2\rho G(z)^T(z - z^*) + \rho^2 \|G(z)\|_2^2 \\
\leq -\rho(2 - \rho)\|G(z)\|_2^2 \\
\leq -M\|G(z)\|_2^2
$$

where $M = \rho_{\text{min}}(2 - \rho_{\text{max}})$ (line 3 is firm nonexpansiveness of $G$)

- (1) implies that

$$
M \sum_{k=0}^{\infty} \|G(z^{(k)})\|_2^2 \leq \|z^{(0)} - z^*\|_2^2, \quad \|G(z^{(k)})\|_2 \to 0
$$

- (1) implies that $\|z^{(k)} - z^*\|_2$ is nonincreasing; $z^{(k)}$ bounded

since $\|z^{(k)} - z^*\|_2$ is nonincreasing, the limit $\lim_{k \to \infty} \|z^{(k)} - z^*\|_2$ exists
since the sequence $z^{(k)}$ is bounded, it has a convergent subsequence

let $\bar{z}_k$ be a convergent subsequence with limit $\bar{z}$; by continuity of $G$,

$$0 = \lim_{k \to \infty} G(\bar{z}_k) = G(\bar{z})$$

hence, $\bar{z}$ is a zero of $G$ and the limit $\lim_{k \to \infty} \|z^{(k)} - \bar{z}\|_2$ exists

let $\bar{z}_1$ and $\bar{z}_2$ be two limit points; the limits

$$\lim_{k \to \infty} \|z^{(k_j_1)} - \bar{z}_1\|_2, \quad \lim_{k \to \infty} \|z^{(k_j_2)} - \bar{z}_2\|_2$$

exist, and subsequences of $z^{(k)}$ converge to $\bar{z}_1$, resp. $\bar{z}_2$; therefore

$$\|\bar{z}_2 - \bar{z}_1\|_2 = \lim_{k \to \infty} \|z^{(k)} - \bar{z}_1\|_2 = \lim_{k \to \infty} \|z^{(k)} - \bar{z}_2\|_2 = 0$$
References

Douglas-Rachford method, ADMM, Spingarn’s method


**image deblurring:** the example is taken from
Primal-Dual Hybrid Gradient Methods

Consider

\[(P) \quad \min_{x \in \mathbb{R}^n} \quad f_1(Bx) + f_2(x)\]

- Saddle point form by Fenchel duality

\[(PD) \quad \min_{x \in \mathbb{R}^n} \sup_{y} \quad -f_1^*(y) + \langle y, Bx \rangle + f_2(x)\]

- The dual problem is

\[(D) \quad \max_{y} \quad -f_1^*(y) - f_2^*(-B^T y)\]

- Saddle point form by introducing new variable \(z = Bx\) in \(P\)

\[\max_{y} \min_{x, z \in \mathbb{R}^n} \quad f_1(z) + f_2(x) + \langle y, Bx - z \rangle\]

- Saddle point form by introducing new variable \(p = B^T y\) in \(D\).

\[\max_{y} \min_{y, p} \quad -f_1^*(y) - f_2^*(p) + \langle x, -B^T y - p \rangle\]
Primal-Dual method from (PD)

Saddle point form by Fenchel duality

\[
(PD) \quad \min_{x \in \mathbb{R}^n} \sup_y \quad -f_1^*(y) + \langle y, Bx \rangle + f_2(x)
\]

- Primal-dual hybrid Gradient (PDHG) Method

\[
x^{k+1} = \arg \min_x \quad \langle y^k, Bx \rangle + f_2(x) + \frac{1}{2\alpha_k} \|x - x^k\|^2_2
\]
\[
y^{k+1} = \arg \max_y \quad -f_1^*(y) + \langle y, Bx^{k+1} \rangle - \frac{1}{2\beta_k} \|y - y^k\|^2_2
\]

- Interpret PDHG as a primal-dual proximal point method for finding a saddle point of (PD)

- Convergence: Esser-Z-Chan, 2010; He-Yuan, 2010; Bonettini-Ruggiero
Modified PDHG

Saddle point form by Fenchel duality

\[
(PD) \quad \min_{x \in \mathbb{R}^n} \sup_y \quad -f_1^*(y) + \langle y, Bx \rangle + f_2(x)
\]

- Modified Primal-dual hybrid Gradient (PDHG) Method

\[
x^{k+1} = \arg \min_x \quad \langle 2y^k - y^{k-1}, Bx \rangle + f_2(x) + \frac{1}{2\alpha_k} \|x - x^k\|_2^2
\]

\[
y^{k+1} = \arg \max_y \quad -f_1^*(y) + \langle y, Bx^k \rangle - \frac{1}{2\beta_k} \|y - y^k\|_2^2
\]

- Esser-Z-Chan, 2010; A. Chambolle and T. Pock, 2010