

# Project 1 for “Algorithms for Big-Data Analysis”

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## 1 Submission Requirement

1. Prepare a report including
  - detailed answers to each question
  - numerical results and their interpretation
2. The programming language can be either matlab, Python or c/c++.
3. Pack all of your codes named as "proj1-name-ID.zip" and upload the file to <https://file.admin.cluster-bicmr.com/u/d/436c91ee1e4f4dac808d/>  
作业提交需要统一打包成压缩文件，命名格式为：proj1-学号-姓名，文件类型随意。文件名中不要出现空格，最好不要出现中文。
4. 请勿大量将代码粘在报告中，涉及到实际结果需要打表或者作图，不要截图或者直接从命令行拷贝结果。
5. 提交word的同学需要提供word原文件并将其转换成pdf文件。
6. If you get significant help from others on one routine, write down the source of references at the beginning of this routine.

## 2 Algorithms for $\ell_1$ minimization

Consider the problem

$$(2.1) \quad \min_x \mu \|x\|_1 + \|Ax - b\|_2,$$

where  $A \in \mathbb{R}^{m \times n}$  and  $b \in \mathbb{R}^m$  are given. Test data are as follows:

```
n = 1024;  
m = 512;  
A = randn(m, n);  
u = sprandn(n, 1, 0.1);
```

```
b = A*u;  
mu = 1e-2;
```

See [http://bicmr.pku.edu.cn/~wenzw/bigdata/Test\\_BP.m](http://bicmr.pku.edu.cn/~wenzw/bigdata/Test_BP.m)

1. Solve (2.1) using CVX by calling different solvers mosek or gurobi.  
CVX, Mosek and Gurobi are available free at:  
CVX: <http://cvxr.com/cvx/>  
Mosek: <http://www.mosek.com/>  
Gurobi: <http://www.gurobi.com/>
2. Solve (2.1) by calling mosek or gurobi directly.
3. Write down and implement one of the following algorithms in Matlab/Python:
  - (a) Classical Augmented Lagrangian method (or Bregman method), where each augmented Lagrangian function is minimized by using the proximal gradient method  
Reference: Wotao Yin, Stanley Osher, Donald Goldfarb, Jerome Darbon, *Bregman Iterative Algorithms for  $l_1$ -Minimization with Applications to Compressed Sensing*
  - (b) Classical Augmented Lagrangian method (or Bregman method), where each augmented Lagrangian function is minimized by using the accelerated proximal gradient method (FISTA or Nesterov's method)  
Reference on FISTA: Amir Beck and Marc Teboulle, *A fast iterative shrinkage thresholding algorithm for linear inverse problems*
4. Write down and implement one of the following algorithms in Matlab/Python:
  - (a) Alternating direction method of multipliers (ADMM) for the primal or dual problem  
Reference: Junfeng Yang, Yin Zhang, *Alternating direction algorithms for  $l_1$ -problems in Compressed Sensing*, SIAM Journal on Scientific Computing, <https://epubs.siam.org/doi/abs/10.1137/090777761>
  - (b) Alternating direction method of multipliers with linearization for the primal or dual problem  
Reference: Junfeng Yang, Yin Zhang, *Alternating direction algorithms for  $l_1$ -problems in Compressed Sensing*, SIAM Journal on Scientific Computing, <https://epubs.siam.org/doi/abs/10.1137/090777761>
5. Requirement:
  - (a) The interface of each method should be written in the following format  

```
[x, out] = method_name(x0, A, b, mu, opts);
```

  
Here, x0 is a given input initial solution, A and b are given data, opts is a struct which stores the options of the algorithm, out is a struct which saves all other output information.
  - (b) Compare the efficiency (cpu time) and accuracy (checking optimality condition) in the format as  
[http://bicmr.pku.edu.cn/~wenzw/bigdata/Test\\_BP.m](http://bicmr.pku.edu.cn/~wenzw/bigdata/Test_BP.m)

### 3 Algorithms For Sparse Inverse Covariance Estimation

Let  $S^n = \{X \in \mathbb{R}^{n \times n} \mid X^T = X\}$ . Let  $S \in S^n$  be a given observation of covariance matrix.

1. Consider the model

$$(3.1) \quad \max_{X \succeq 0} \log \det X - \text{Tr}(SX) - \rho \|X\|_1,$$

where  $\|X\|_1 = \sum_{ij} |X_{ij}|$ .

- (a) data sets: set  $n = 30$ , generate models 1 and 2 in section 5.1 of page 599 at <http://www-stat.wharton.upenn.edu/~tcai/paper/Precision-Matrix.pdf>
- (b) Derive the dual problem of (3.1).
- (c) Solve (3.1) using CVX using a few  $\rho$ , for example, 10, 0.1, 0.001.
- (d) Write down and implement a first-order type algorithm for solving (3.1) with the same  $\rho$  in (b).

2. (Optional, Extra-credit) Consider the model

$$(3.2) \quad \min_{X \succeq 0} \|X\|_1, \text{ s.t. } \|SX - I\|_1 \leq \sigma,$$

where  $I$  is the identity matrix. The test data is the same as 1(a).

- (a) Derive the dual problem of (3.2).
- (b) Solve (3.2) using CVX using a few  $\sigma$ , for example, 10, 0.1, 0.001.
- (c) Solve (3.2) with the same  $\sigma$  in (b) by calling SDPT3 directly.  
SDPT3: <http://www.math.nus.edu.sg/~mattohkc/sdpt3.html>
- (d) Write down and implement a first-order type algorithm for solving (3.2) with the same  $\sigma$  in (b).

Note that there is a constraint  $X \succeq 0$  in both (3.1) and (3.2).