Homework 7 for “Algorithms for Big-Data Analysis”

March 12, 2018

Note: Please write up your solutions independently. If you get significant help from others, write down the source of references. A formal mathematical proof for all your claims is required.

1. Given a $n \times p$ matrix $X$. Derive the optimal solution for the following problem:

$$\min_{Z,V} \|X - ZV\|_F^2, \quad \text{s.t.} \quad V^TV = I, Z^T1 = 0,$$

where $Z$ is a $n \times q$ matrix and $V$ is a $q \times p$ matrix.

2. Derive the dual optimization problem for

$$\min_{w,b,\xi} \frac{1}{2} \|w\|_2^2 + C_1 \sum_{i=1}^n \xi_i + C_2 \sum_{i=1}^n \xi_i^2$$

$$\text{s.t.} \quad y_i \cdot (x_i \cdot w + b) \geq 1 - \xi_i, \forall i = 1, \ldots, n$$

$$\xi_i \geq 0, \forall i = 1, \ldots, n$$

3. Properties of Submodular Functions

(a) Prove that any non-negative submodular function is also subadditive, i.e. if $F : 2^X \to \mathbb{R}_+$ is submodular then $F(S \cup T) \leq F(S) + F(T)$ for any $S, T \subseteq X$. Here, $\mathbb{R}_+ = \{x \in \mathbb{R}, x \geq 0\}$.

(b) Prove that a function $F : 2^X \to \mathbb{R}_+$ is submodular if and only if for any $S, T \subseteq X$, the marginal contribution function $F_S(T) = F(S \cup T) - F(S)$ is subadditive.

4. Given finite ground set $X$, and given $w_d \in [0, 1]$ for all $d \in X$, define

$$F(S) = \prod_{d \in S} w_d,$$

where $F(\emptyset) = 1$. Is this submodular, supermodular, modular, or neither?