Sketch of Thm A + B

<u>Key ingredients</u>: (a). Slope filtration Theorem

(b). Berger's construction of modification

(a) is already explained in Ji Yibo's talk.

(b): D : φ-module over $B^\dagger_{rig,K}$. Then

$$\{B^\dagger_{rig,K}\text{-lattice } D' \text{ of } D,\ \varphi^* D' \simeq D\}$$

$$\left\{\begin{array}{l}\varphi\text{-compatible lattices } \{M_n\} \text{ s.t.} \\ M_n[\tfrac{1}{t}] = D^{r_n} \otimes_{B^{\dagger, r_n}_{rig,K},\, \iota_n} K_n((t))\end{array}\right\}$$

<u>Proof of Thm A:</u> W : filtered φ-module of dim d. Let $\hat{D} = W \otimes B^\dagger_{rig,K}$

Then $M_n = \mathrm{Fil}^0 (W \otimes_K K_n((t)))$ are φ-compatible lattices

Thus (b) gives rise to a (φ, Γ)-module D s.t.

$$D[\tfrac{1}{t}] = \hat{D}[\tfrac{1}{t}]$$

Then $\operatorname{rank} D = d$, and $(D[\frac{1}{t}])^\Gamma = W$

It follows that for any saturated (φ,Γ)-submodule D' of D

$$\dim (D'[\tfrac{1}{t}])^\Gamma = \operatorname{rank} D'$$

Conversely, for any φ-submodule $W' \subset W$. by (b)

$\{\mathrm{Fil}^0 (W \otimes_K K_n((t))) \}_{n \geq 0}$ gives rise to a saturated

(φ,Γ) - submodule of D. Hence we have

$$\{\text{saturated } (\varphi,\Gamma)\text{-submodule}\} \leftrightarrow \{\varphi\text{-submodule of } W\}$$

Now use the condition that $(t_N - t_H)(D') = \deg(W')$

we see that first, $\deg(D) = 0$.

Second, W is weakly admissible $\Leftrightarrow$ D is semi-stable as a φ-module over $B_{rig}^{\dagger,K}$

Thus by slope filtration theorem, W weakly admissible $\Rightarrow$ D is étale $\Rightarrow$ the desired Galois repn.

(See Yu Jiahong's note for details)

<u>Proof of Thm B:</u> Now suppose V is a deRham repn.

$$D = D_{rig}^{\dagger}(V) \quad W = D_{dR}(V)$$

This time we use $\{W \otimes_K K_n[[t]]\}_{n \geq 0}$ as the φ-compatible lattices (here we use $D_{dR} = D_{dif}^{\Gamma}$ to relate it to localization of D)

to get a (φ,Γ)-module $\tilde{D}$.

<u>Remark:</u> Actually here $(D,\tilde{D})$ is the same as $(D,\hat{D})$ in the proof of Thm A.

<u>upshot:</u> $\nabla(\hat{D}) \subseteq t\hat{D} \Rightarrow \hat{D}$ is a p-adic differential equation

with Frobenius structure.

Thus by p-adic monodromy theorem in p-adic differential equation (which can be proved by slope filtration theorem & Tsuzuki's result). (see Theorem 8.1 of Yu Jiahong's note)

We conclude.

<u>Exercise:</u> In the proof of Thm A, for general w, find an algorithm to determine the slope polygon of D.