Additional knowledge

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Cramér-Rao inequality

A mapping $\hat{\xi} = [\hat{\xi}^1, \cdots, \hat{\xi}^n] : \mathcal{X} \to R^n$ is called an estimator. If $E[\hat{\xi}(X)] = \xi$ for $\forall \xi \in \Xi$, we say that $\hat{\xi}$ is an **unbiased estimator**. The variance-covariance matrix $V_\xi[\hat{\xi}] = [v_\xi^{ij}]$ is defined as

$$v_\xi^{ij} \triangleq E_\xi[(\hat{\xi}^i(X) - \xi^i)(\hat{\xi}^j(X) - \xi^j)]$$

Theorem (Cramér-Rao inequality)

The variance-covariance matrix $V_\xi[\hat{\xi}]$ of an unbiased estimator $\hat{\xi}$ satisfies $V_\xi[\hat{\xi}] \geq G(\xi)^{-1}$.

Cramér-Rao inequality

Theorem

Let $I(\theta) \triangleq \int (\frac{d\ln f(x,\theta)}{d\theta})^2 f(x,\theta)dx > 0$. *If* $\psi(X_1, \cdots, X_n)$ *is an unbiased estimator of* $g(\theta)$, *then we have*

$$Var_\theta(\psi(X_1, \cdots, X_n)) \geq \frac{[g'(\theta)]^2}{nI(\theta)}.$$

This accomodates with the former theorem with 1-dimension and $g(\theta) = \theta$.

Surface

A (topological) **surface** is a topological space in which every point has an open neighbourhood homeomorphic to some open subset of the Euclidean plane E^2. A **closed surface** is a surface that is compact and without boundary.

Euler characteristic

On polyhedra, The Euler characteristic $\chi = V - E + F$, where V, E, and F are respectively the numbers of vertices (corners), edges and faces in the given polyhedron.

Surface admitting flat connection

Any nonclosed surface admits a flat torsionfree affine connection, and closed surfaces of zero euler characteristic and only them, admit flat torsionfree connection.