

PUBLICATION LIST

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1. RATIONALLY CONNECTED VARIETIES

- (1) (with Amit Hogadi) *Degenerations of Rationally Connected Varieties*. *Trans. Amer. Math. Soc.* **361** (2009), no. 7, 3931–3949.

We give an affirmative answer to the following question raised by Kollár: Do the degenerations of rationally connected varieties defined over a field k ($\text{char}(k) = 0$ and k may not be algebraic closed) always contain a rationally connected subvariety (by definition, it says in particular that the subvariety is absolute irreducible).

In an early paper, Kollár gave a partial answer to this question, namely, he proved that a degeneration of Fano varieties contains an absolute irreducible subvariety over k . Combining his idea with Hacon-Mckernan’s extension theorem, we first prove the theorem for the cases when the general fibers are Fano varieties or the relative dimension is 0. Then we use the results from the minimal model program (MMP), which is proved by Birkar, Cascini, Hacon and Mckernan, to reduce the general case to these 2 special cases.

- (2) *Notes on π_1 of Smooth Loci of Log Del Pezzo Surfaces*. *Michigan Math. J.* **58** (2009), no. 2, 489–515.

It is known that the fundamental groups of smooth loci of Log del Pezzo surfaces are finite groups. In this note, based on the classification results of Dolgachev and Iskovskikh on finite subgroups of Cremona groups, we study these finite groups. A short table containing these groups is given. And lots of groups on the table are proved to be fundamental groups.

- (3) *Strong Rational Connectedness of Surfaces. J. Reine Angew. Math.* **665** (2012), 189-205.

For a nonproper smooth variety defined over \mathbb{C} , it is asked whether the rational connectedness implies the strong rational connectedness. We discuss this question in the case of surfaces. And we prove in lots of situations, including the smooth locus of a log del Pezzo surface, whose rational connectedness is proved by Keel-McKernan, the rationally connected surface is indeed strongly rationally connected. This confirms a conjecture due to Hassett and Tschinkel. The main idea we use here is to investigate the rational curve theory of the canonical smooth separated cover stack of the compactification surface. The natural frame is the n -pointed twisted stable map theory with the target space a Deligne-Mumford stack, which is constructed by Abramovich-Vistoli. Then we study the deformation theory there.

- (4) *Weak Approximation for Low Degree del Pezzo Surfaces. J. Algebraic Geom.* **21** (2012), no. 4, 753-767.

Applying the main theorem of the above paper, we verify the weak approximation conjecture of Hassett-Tschinkel for del Pezzo surfaces of degree 1 defined over the function field of curve $K(C)$, which can be completed to a ‘generic family’ over C . The approach follows from idea of Hassett-Tschinkel: we get a section with prescribed jets by adding appropriate ‘teeth’ to an arbitrary section and deform it.

- (5) (with János Kollár) *Fano Varieties with Large Degree Endomorphisms.* 2 pages. arXiv:0901.1692

In this unpublished note, we give examples of Fano varieties X with Picard number 1, which have terminal singularities and admit endomorphisms with degree larger than 1. This is a counterexample to a conjecture of De-Qi Zhang. However, if we assume X is smooth, the long standing conjecture says that X is indeed \mathbb{P}^n .

2. SINGULARITIES

- (6) *Finiteness of algebraic fundamental groups. Compos. Math.* **150** (2014), Issue 03, 409-414.

Applying the local-to-global induction, especially by considering the *Kollár component*, we prove that the algebraic fundamental group of a klt singularity is finite.

- (7) (with Tommaso de Fernex, János Kollár) *The dual complex of singularities.* To appear in *Proceedings of the conference in honor of Yujiro Kawamata's 60th birthday, Advanced Studies in Pure Mathematics.*

By investigating when the MMP process does not change the homotopic class of the dual complex of a dlt pair, we prove there is a well defined up to PL homeomorphism topological space which is a representative of the

dual complex of a \mathbb{Q} -Cartier isolated singularity. Using the same idea, we also show that the dual complex of a klt singularity as well as the degeneration of rationally connected varieties is contractible.

- (8) (with Johannes Nicaise) *Poles of maximal order of motivic zeta function. Duke Math. J.* **165** (2016), Issue 2, 217-243.

We prove a conjecture of Veys which says that the only maximal order pole of zeta functions is the opposite of the log canonical threshold. We prove this by running MMP and analyze how the dual complex changes in the process. So we prove a stronger geometric statement which says that the only candidate of pole with maximal order is the opposite of the log canonical threshold.

- (9) Motivic zeta function via dlt modification. *Michigan Math. J.* **65** (2016), Issue 1, 89-103.

We invent a notion called dlt motivic zeta function using the dlt modification and show it does not depend on the choice of the dlt modification.

- (10) (with János Kollár) The dual complex of Calabi-Yau pairs. Accepted by *Invent. Math.*.

We show that for a Calabi-Yau pair (X, D) the fundamental group of the dual complex of D is a quotient of the fundamental group of the smooth locus of X if $\dim D(D) \geq 2$, hence its pro-finite completion is finite. This is achieved by running a carefully chosen MMP to change the model such that D supports a large divisor. And then the Lefschetz hyperplane theorem for open varieties connects the fundamental group of the smooth locus of X and the fundamental group of the divisor.

- (11) (with Zhiyu Tian) Finiteness of fundamental groups.

We use stratified Morse theory to refine the argument in the study of (6), and show that the finiteness of the fundamental group of the smooth locus of $n - 1$ dimensional log Fano varieties implies the same thing for the local fundamental group of n -dimensional klt singularities.

- (12) (with Chi Li) Stability of Valuations and Kollár Components

We use the construction of Kollár component in (6) to study the minimizer of the normalized volume function, especially the case of rational rank 1. We show that if there is K-semistable Kollár component, then it is the minimizer, and it's the only one among all Kollár components. This study clarifies the quasi-regular case, when the entire subject of studying the normalized volume function aims at giving a picture similar to Sasakian geometry, but for any klt singularity.

3. KSBA MODULI SPACES AND MMP

- (13) (With Gueorgui Todorov) *On Effective Log Iitaka Fibration for 3-folds and 4-folds. Algebra Number Theory* **3** (2009), no. 6, 697–710.

We study the effective log Iitaka fibration problem for a *klt* pair (X, Δ) , where the coefficients of Δ are in a DCC set \mathcal{A} . We prove for 3-folds and 4-folds, if the log Kodaira codimension is 2, then there is a uniform $N = N(\mathcal{A})$ such that $\lfloor N(K_X + \Delta) \rfloor$ gives the Iitaka fibration. We in fact first prove this result in the surface case following Alexeev's work on boundness of log surfaces. Then applying Kawamata's canonical bundle formula and Fujino-Mori's result, we get a bound of the denominators of the moduli part. Then when the base is curve or surface, this is enough for us to get the result by analysing the adjoint bundle on them.

- (14) (with Christopher Hacon, James M^cKernan) *On the Birational Automorphisms of Varieties of General Type. Annals of Math.* **177** (2013), Issue 3, 1077-1111.

This is the first paper of our project which aims to study the boundedness of volumes for singular pairs with arbitrary dimension. In particular, we prove the following result: the order of the automorphism group of a variety of general type X is bounded by $N \cdot \text{vol}(K_X)$ where N is a constant depending only on the dimension of X . This previously was only known for surfaces. The first step of the proof is to show the birational boundedness of the underlying space for global quotient pairs with volumes bounded above. The remaining parts of the proof work for general log canonical pairs, whose coefficients are in a fixed DCC set. Hence they will be used in a sequel, which is trying to give an affirmative answer to a Kollár's conjecture.

- (15) (with Christopher Hacon, James M^cKernan) *ACC for log canonical thresholds. Ann. of Math.* **180** (2014), Issue 2, 523-571.

In this paper, we continue our investigation on the boundedness behavior of volumes of singular pairs in (14). We verify Kollár's following conjecture: if we fix a positive integer n and a DCC set \mathcal{A} , then the set of volumes of n -dimensional log canonical pairs (X, Δ) with $\Delta \in \mathcal{A}$ satisfies DCC. Besides the ingredients appearing in the first paper, there are two more steps. We first prove Kollár's conjecture for the case that \mathcal{A} is a finite set of rational numbers. Then we combine the remaining of the proof with the proof of global ACC conjecture for numerically trivial pairs. In particular, in the course of proving Kollár's conjecture, we also establish the argument for the ACC conjecture for numerically trivial pairs. And it is straightforward (and was known before) that the latter one implies Shokurov's ACC conjecture on log canonical thresholds.

- (16) (with Christopher Hacon) *Existence of log canonical closures. Invent. Math.* **192** (2013), Issue 1, 161-195.

We prove the following result on the existence of good models: Let $(X, \Delta)/U$ be a proper dlt pair such that over a nonempty open set $U^0 \subset U$, $(X^0, \Delta^0) = (X, \Delta) \times_U U^0$ has a good model, and any lc center of (X, Δ) has its image meeting U^0 , then (X, Δ) has a good model. The result

seems to be technical, but it indeed has many interesting consequences including the existence of lc flips, the properness of the moduli spaces of stable varieties the existence of lc closure and so on.

- (17) (with Christopher Hacon, James M^cKernan) Boundedness of moduli of varieties of general type.

We finish the project in (14) and (15) and show that the moduli space of KSBA stable pairs is bounded. The key point here is to show that in a log smooth log canonical family, if one fiber has a good minimal model, then so does any other fiber. We rely on the same result on klt case proved using analytic tools.

- (18) (with Christopher Hacon) *On Finiteness of \mathbf{B} -representation and Semi-log Canonical Abundance*. To appear in *Minimal models and extremal rays- Proceedings of the conference in honor of Shigefumi Moris 60th birthday, Advanced Studies in Pure Mathematics*.

We show the finiteness of \mathbf{B} -representation. Using it we show that slc abundance is implied by lc abundance. This is supposed to be one step to prove the general abundance conjecture. In fact, with this result, we verify abundance conjecture in special cases.

- (19) (with Yuji Odaka) *Log-canonical modification of singular pairs and its applications*. *Math. Res. Lett.* **19** (2012), no. 2, 325–334.

Using the main result of (16), we show the existence of log canonical modification. Then with Kollár’s gluing result, we generalize our result to the existence of semi-lc modification. As an application, this removes the conjectural assumption on MMP of Odaka’s result saying that a polarized scheme is K -stable, only if it is slc.

- (20) (with Xiaowei Wang) *Nonexistence of asymptotic GIT compactification*. *Duke Math. J.* **163** (2014), Issue 12, 2217–2241.

We show that if a KSBA limit of a family of smooth canonically polarized varieties is not asymptotic GIT Chow semistable then this family does not yield any asymptotic GIT Chow semistable limit. This answers a longstanding problem. We show that the semi-stable filling minimizes the GIT height and the KSBA limit minimizes the Donaldson-Futaki invariant. Since the latter is the limit of the formers, if there is a asymptotic GIT Chow semistable limit, then it must be identical to the KSBA limit.

- (21) (with Zsolt Patakfalvi) *Ampleness of the CM line bundle on the moduli space of canonically polarized varieties*. Accepted by *Algebraic Geometry*.

We show that the CM line bundle on the compact KSBA moduli space is always ample. CM line bundle it is the leading term of the Mumford-Knudson construction for the determinant of the push forward of the multiples of the relative canonical bundle, which is ample by Kollár’s result.

- (22) (with Christopher Hacon) *Boundedness of log Calabi-Yau pairs of Fano type*. Accepted by *Math. Res. Lett.*

We show that the klt log Calabi-Yau pair with big boundary whose coefficients are in a DCC set indeed form a bounded family, using the boundedness result for log general type pairs in (15).

4. MMP IN POSITIVE CHARACTERISTIC

- (23) (with Christopher Hacon) *On the three dimensional minimal model program in positive characteristic. J. Amer. Math. Soc.* **28** (2015) no. 3, 711-744.

We show that for a 3-fold extremal dlt flipping contraction defined over an algebraically closed field of characteristic $p > 5$, such that the coefficients of the boundary are in the standard set $\{1 - \frac{1}{n} | n \in \mathbb{N}\}$, then the flip exists. As a consequence, we prove the existence of minimal models for any projective \mathbb{Q} -factorial terminal variety X with pseudo-effective canonical class. We obtain this result using the original Shokurov's argument in characteristic 0, but then replace many characteristic 0 techniques by the recent characteristic p tools developed in the F -singularity theory. Especially, we replace Kawamata-Viehweg vanishing theorem by only looking at the image of absolute Frobenius. Then we prove in the case we consider, it is the same as the complete linear system.

- (24) (with Paolo Cascini and Hiromu Tanaka) *On base point freeness in positive characteristic. Ann. Sci. École Norm. Sup.* **48** (2015) no. 5, 1239-1272..

We apply a new approach in positive characteristic to cutting subschemes, from which we can extend sections of the line bundle of an adjoint form. As a result, we prove for any F -regular pair (X, B) and an ample divisor A , the \mathbb{Q} -divisor $K_X + B + A$ is ample if it is strictly nef and is big if it is nef and of maximal nef dimension. These are corollaries of standard base point free and cone theorems. Then we also achieve new results in 3-dimensional MMP in positive characteristic.

- (25) *On base point free theorem of threefolds in positive characteristic. J. Inst. Math. Jussieu* **14** (2015) Issue 3, 577-588.

We prove that if (X, Δ) is a three dimensional klt pair over a field k with $\text{char}(k) > 5$, then base point free theorem holds for a big and nef line bundle L such that $L - K_X - \Delta$ is big and nef.

5. RATIONAL POINTS

- (26) (with Hélène Esnault) *Congruence for rational points over finite fields and coniveau over local fields. Trans. Amer. Math. Soc.* **361** (2009), no. 5, 2679-2688.

This paper contains two parts. In the first part, we prove that for a projective smooth variety X defined over a local field K , if its ℓ -adic cohomology is supported on a codimensional 1 subvariety, then every model over the integers of K has a k -rational point, where k is the finite residue

field of K . In an early paper, Esnault gave a proof when K is a p -adic field. And the argument here is similar, but we have to deal with the inseparable extension case. If the model \mathcal{X} is regular, then a stronger result was also proved by Esnault before, namely, one has a congruence $|\mathcal{X}(k)| \equiv 1$ modulo $|k|$ for the number of k -rational points. The second part of this paper shows by a example that without the regularity assumption, this stronger result fails.

6. BERKOVICH SPACE

- (27) (with Johannes Nicaise) *The essential skeleton of a degeneration of algebraic varieties*. To appear in *Amer. J. Math.*.

We explore the connection between minimal model program and Berkovich space. As a result, we identify the essential skeleton of the Berkovich space for a variety with semiample canonical bundle and the dual complex constructed for the reduced special fiber of a dlt minimal model.

7. FANO MODULI

- (28) (with Chi Li) *Special test configurations and K -stability of Fano varieties*. *Annals of Math.* **180** (2014), Issue 1, 197-232.

Let X be a Fano manifold. For any given test configuration $(\mathcal{X}, \mathcal{L})$ of $(X, -rK_X)$, we make a few modifications rooted in the minimal model program to simplify the test configuration. More precisely, we first take the log canonical modification and then use the minimal model program (MMP) with scaling. Finally, we resolve again and carefully run another MMP such that at the end, the special fiber becomes a klt Fano varieties. We show that the Donaldson-Futaki invariant is always non-increasing along the process. This implies that, when X is Fano, to test K -(semi)stability, we only need to test on the special test configurations. We also show by a counter-example that the ‘right’ definition of K -(poly)stability should only involve normal test configurations.

- (29) (with Chi Li and Xiaowei Wang) *Degeneration of Kähler-Einstein Fano manifolds*.

We show that there is a proper algebraic space which is the coarse moduli space of all smoothable K -semistable Fano varieties. Pointwisely it parameterizes all smooth K -polystable Fano varieties. We first show the uniqueness of the degeneration of smooth KE Fano manifolds to a K -polystable Fano by using the solution of Yau-Tian-Donaldson conjecture due to Chen-Donaldson-Sun and Tian, and an investigation of the geometry orbit spaces. Then we combine it with the general theory of the existence of good quotient space of a stack, and verify all the needed assumptions.

(30) (with Chi Li and Xiaowei Wang) Quasi-projectivity of the moduli space of smooth Kähler-Einstein Fano manifolds.

Continuing the work of (29), we show that the restriction of CM line bundle on the open locus parametrizing all smooth KE Fano manifolds indeed gives an ample line bundle.