

# 2025全国几何分析年会

WORKSHOP ON GEOMETRIC ANALYSIS 2025

## 会议日程

6 月 30 日（星期一）上午				
9:00-9:50	汪徐家	西湖大学	Topological methods for some nonlinear geometric and PDE problems	主持人：张伟平
10:00-10:50	马世光	南开大学	Isolated singularities of solutions of certain quasi-linear elliptic inequalities	主持人：戴先哲
10:50-11:10	茶歇			
11:10-12:00	黄耿耿	复旦大学	Monge-Ampere equations in convex polytopes	主持人：刘佳堃
12:00-13:30	午餐			
2025 年 6 月 30 日（星期一）下午				
14:00-14:50	黄显涛	中山大学	Ricci curvature and the fundamental group	主持人：戎小春
15:00-15:50	熊金钢	北京师范大学	An improved Aleksandrov estimate and an associated variational problem	主持人：陆志勤
15:50-16:10	茶歇			
16:10-17:00	靳晓尚	华中科技大学	Half generic heat dispersions law on compact Riemannian manifolds	主持人：苏广想
19:30-21:30	晚餐			
2025 年 7 月 1 日（星期二）上午				
9:00-9:40	开幕式 & 合影			
9:50-10:40	丁剑	北京大学	Geometric properties of random walks	主持人：朱熹平
10:40-11:00	茶歇			
11:00-11:50	安歆亮	National University of Singapore	On the mathematical theory of gravitational collapse	主持人：张晓
12:00-13:30	午餐			

2025 年 7 月 1 日（星期二）下午				
14:00-14:50	刘磊	华中师范大学	Compactness problem related to Sacks-Uhlenbeck harmonic maps	主持人：陈竞一
15:00-15:50	林龙智	University of California, Santa Cruz, USA	Energy convexity and uniformity of H-surface flow in two dimensions	主持人：张会春
15:50-16:10	茶歇			
16:10-16:40	乔钰翔	北京大学	Sharp $L^\infty$ estimates for fully non-linear elliptic equations on compact complex manifolds	主持人：关波
16:50-17:20	汪铃	北京大学	Monge-Ampère type equations in two dimensions	主持人：关波
17:30-19:30	晚餐			
2025 年 7 月 2 日（星期三）上午				
9:00-9:50	周鑫	Cornell University, USA	Area/Volume spectrum and its applications in geometric variational problems	主持人：韦国芳
10:00-10:50	韩冀原	西湖大学	On H-minimizer of a Fano fibration	主持人：刘钢
10:50-11:10	茶歇			
12:00-13:30	午餐			
2025 年 7 月 2 日（星期三）下午				
14:00-17:30	自由讨论			
17:30-19:30	晚餐			
2025 年 7 月 3 日（星期四）上午				
9:00-9:50	袁域	University of Washington, USA	Constant rank theorem for special Lagrangian and quadratic Hessian equations	主持人：刘小博
10:00-10:50	郭常予	山东大学	Improved geometric characterization of Gromov hyperbolicity in Euclidean spaces and beyond	主持人：冯惠涛
10:50-11:10	茶歇			
11:10-12:00	周德堂	Universidade Federal Fluminense, Brazil	Laplacian spectrum on solitons	主持人：张旗
12:00-13:30	午餐			

# 2025全国几何分析年会

WORKSHOP ON GEOMETRIC ANALYSIS 2025

2025 年 7 月 3 日（星期四）下午				
14:00-14:50	姜旭旻	大湾区大学	Asymptotic behavior of minimal surfaces in hyperbolic space	主持人：盛为民
15:00-15:50	马飙	北京大学	Constructing stable Hilbert bundles with irrational slopes	主持人：刘博
15:50-16:10	茶歇			
16:10-16:40	姚萱	Cornell University, USA	On the topology of the manifolds with positive intermediate curvature	主持人：李海中
16:50-17:20	刘亚雄	University of Maryland, USA	Existence of weighted cscK metrics	主持人：李海中
17:30-19:30	晚餐			
2025 年 7 月 4 日（星期五）上午				
8:30-9:20	方浩	University of Iowa, USA	Gårding Polynomials and its applications	主持人：蒋美跃
9:30-10:00	顾坪昕	清华大学	Weinstock inequality in hyperbolic space	主持人：葛化彬
10:10-10:40	李希伦	北京大学	Parabolic Hermitian-Einstein metrics over nonKahler manifolds	主持人：葛化彬
10:40-11:00	茶歇			
11:00-11:50	袁原	Syracuse University, USA	Characterization of the complex ball via invariant metrics	主持人：朱小华
12:00-13:30	午餐			
2025 年 7 月 4 日（星期五）下午				
14:00-17:30	自由讨论			
17:30-19:30	晚餐			

# 报告摘要

(按照报告人姓名字母排序)

## 50 分钟报告

### **On the Mathematical Theory of Gravitational Collapse**

安歆亮 (National University of Singapore)

Abstract: In this talk I will present several results studying the mathematical details of gravitational collapse in general relativity. In particular, these results are about Kerr black hole formation, a new approach toward Penrose inequality and weak cosmic censorship.

### **Geometric properties of random walks**

丁剑 (北京大学)

Random walk is one of the few most fundamental models in probability theory. In this talk, I will discuss the geometric properties of random walks as well as a few random-walk-based models.

The talk will be in an overview style, with a review (biased by my personal interest) on some classic and recent results as well as a glimpse into some future directions.

### **Gårding Polynomials and its applications**

方浩 (University of Iowa, USA)

In this talk we report some recent joint work with Biao Ma.

The 1959 works of Gårding on hyperbolic polynomials has been widely used to study geometric fully non-linear PDEs of Hessian type, under the framework of Caffarelli-Nirenberg-Spruck theory.

Generalizing earlier works of C.-M. Lin and some of our previous results, we introduce a class of multi-affine polynomials that is named after Gårding. Gårding polynomials are characterized by their associated cones and include key examples of hyperbolic polynomials. We explore their relations to Branden-Huh's Lorentzian polynomials and derive desired inequalities. Finally, we conclude with some applications in complex geometry and pose some future problems.

## **Improved geometric characterization of Gromov hyperbolicity in Euclidean spaces and beyond**

郭常予 (山东大学)

In a seminal work [Asterisque 2001], Bonk-Heinonen-Koskela developed a rich uniformization theory in metric spaces that extends the classical uniformization theorem of Riemann. Then they made a conjecture about characterization of Gromov hyperbolicity in terms of two geometric conditions. In a following work [Invent. Math. 2003], Balogh and Buckley successfully verified this conjecture for bounded Euclidean domains, building upon the work of Bonk-Heinonen-Koskela.

In this talk, we shall address the conjecture of Bonk-Heinonen-Koskela for general Euclidean domains and present an improvement of the main result of Balogh and Buckley. In particular, we provide both a counter-example to a fundamental open problem asked by Balogh-Buckley and a positive result under a sharp extra geometric assumption. Our proofs are completely new and elementary, allowing for a straightforward extension in doubling metric spaces, which substantially improved the main result of Koskela et al. in [Ann. Sci. Ec. Norm. Super. 2014].

## **On H-minimizer of a Fano fibration**

韩骥原 (西湖大学)

Hamilton-Tian theorem predicts a unique limit (a shrinking soliton) of the Kähler Ricci flow on a Fano variety. H-minimizer plays an important role in characterizing this limit. In a recent work by Song Sun and Junsheng Zhang, the authors show that a complete shrinking soliton is quasi-projective. This result emphasizes the importance to study shrinking solitons on Fano fibrations over affine varieties. In a joint work with Lu Qi and Linsheng Wang, we show that there exists a unique H-minimizer for a Fano fibration germ.

### **Monge-Ampere equations in convex polytopes**

黄耿耿（复旦大学）

In this talk, we will talk about our recent regularity results of Monge-Ampere equations in convex polytopes with Guillemin boundary condition or Dirichlet boundary condition. This is a joint work with Weiming Shen.

### **Ricci Curvature and the Fundamental Group**

黄显涛（中山大学）

In this talk, I will first review the research on the fundamental groups on noncompact manifolds with nonnegative Ricci curvature, then I will introduce a recent joint work with Hongzhi Huang which says that if  $M$  is a 4-dimensional open manifold with nonnegative Ricci curvature, and its universal cover has Euclidean volume growth, then the fundamental group of  $M$  is finitely generated and contains an abelian subgroup whose index is bounded by a universal constant.

### **Asymptotic behavior of minimal surfaces in hyperbolic space**

姜旭旻（大湾区大学）

We talk about the asymptotic behavior of minimal surfaces in hyperbolic space that are asymptotic to a differential manifold at infinity. For the hypersurface case, we elaborate on the existence theorem, rough asymptotic behavior, and regularity theorem. Additionally, we address the case of minimal surfaces with high codimension.

### **Half generic heat dispersions law on compact Riemannian manifolds**

靳晓尚（华中科技大学）

In this talk, we will start with the half capacity law in Euclidean space which describes the equivalence between capacity and Laplace-capacity. Then we extend it to the general  $p$ -capacity case and to the general heat dispersions law on manifolds. As an application, we also obtain the half eigenvalue law for the  $p$ -Laplacian with Robin conditions. This is a joint work with Professor Xiao.

## Energy Convexity and Uniformity of H-Surface Flow in Two Dimensions

林龙智 (University of California, Santa Cruz, USA)

In this talk, we present a convexity property of the energy functional associated with surfaces of prescribed mean curvature (also known as H-surfaces) in  $\mathbb{R}^3$  with prescribed Dirichlet boundary data, leading to a quantitative uniqueness result for solutions to the H-surface equation. We will also discuss an energy convexity property along the heat flow for H-surfaces in  $\mathbb{R}^3$ , assuming only that the initial Dirichlet energy is sufficiently small. This analysis yields a new result establishing the existence of weak solutions, long-time existence, and uniform convergence of the flow to the unique solution of the H-surface system with prescribed Dirichlet boundary conditions. This talk is based on recent joint work with Da Rong Cheng and Xin Zhou.

## Compactness problem related to Sacks-Uhlenbeck harmonic maps

刘磊 (华中师范大学)

In this talk, we will first review some classical (interior) compactness results related to harmonic maps and Sacks-Uhlenbeck harmonic maps. Then we will talk about the free boundary case. Also, we will study some asymptotic properties when the domain metrics degenerate. Roughly speaking, we will establish some (generalized) energy identities in this blow-up process.

## Constructing stable Hilbert bundles with irrational slopes

马飙 (北京大学)

On any complex smooth projective curve with positive genus, we construct Hilbert bundles that admit Hermitian-Einstein metrics with irrational slopes. A main constructive step is by investigating the arithmetic property of the upper half plane in Bridgeland's definition of stability conditions. The main analytic ingredient in our proof is a notion called a well-approximation stable bundle. This notion helps us to apply the Diophantine approximation to Donaldson's functional and bound the  $L^\infty$  norm of Hermitian-Einstein metrics. We further study the continuous structures, smooth structures, and holomorphic structures on such Hilbert bundles. This talk is based on a work joint with Yucheng Liu.

### **Isolated singularities of solutions of certain quasi-linear elliptic inequalities**

马世光（南开大学）

I will talk about our recent work, which completely classifies the asymptotic behavior of isolated singularities for a kind of p-Laplace type differential inequality.

### **Topological methods for some nonlinear geometric and PDE problems**

汪徐家（西湖大学）

Topological methods have been extensively used in nonlinear geometric and PDE problems. In recent years, new topological methods have been introduced in the study of nonlinear geometric partial differential equations. In this talk, I will discuss two new topological methods introduced in my unpublished papers, joint with my collaborators.

### **An Improved Aleksandrov Estimate and an Associated Variational Problem**

熊金钢（北京师范大学）

I will introduce an improved formulation of the Aleksandrov estimate concerning the difference between two convex functions, under the condition that one of the functions is strictly convex and smooth. Furthermore, a corresponding variational problem can be set up. It is demonstrated that extremal functions for this problem exist, and a concentration phenomenon manifests. This is joint with Xushan Tu and Tianling Jin.

### **Constant rank theorem for special Lagrangian and quadratic Hessian equations**

袁域（University of Washington, USA）

We present a constant rank theorem for saddle solutions to special Lagrangian and quadratic Hessian equations (a minimum principle for the minimum eigenvalue of Hessian of a solution to elliptic equations satisfying a relaxed convexity, precisely inverse-convexity condition). The argument also leads to new Liouville type results for the special Lagrangian equations with subcritical phase, matching the known rigidity results for semi-convex entire solutions to the quadratic Hessian equation. This is joint work with W. Jacob Ogden.

## Characterization of the complex ball via invariant metrics

袁原 (Syracuse University, USA)

I will talk about a recent work which characterizes the complex ball among bounded smooth pseudoconvex domains via various invariant metrics.

## Laplacian spectrum on solitons

周德堂 (Universidade Federal Fluminense, Brazil)

In this talk, I will report some of our results about the spectrum of Laplacian and drift Laplacian on Ricci shrinkers, Self shrinkers, Ricci expanders and self expanders during the recent years. When the spectrum is discrete, we will consider the estimate of the eigenvalues and their rigidities.

The results are based on the joint works with my coworkers: Xu Cheng, Helton Leal, Matheus Vieira and Franciele Conrado.

## Area/Volume Spectrum and Its Applications in Geometric Variational Problems

周鑫 (Cornell University, USA)

This talk will explore several definitions of the area/volume spectrum and focus on its role in geometric variational theory. Specifically, we will discuss applications of the spectrum to existence problems for minimal surfaces and constant mean curvature surfaces. Furthermore, we will present some recent research advances concerning the area/volume spectrum in prescribed mean curvature surface theory.

## 30 分钟报告

## Weinstock inequality in hyperbolic space

顾坪昕 (清华大学)

In this talk, we establish the Weinstock inequality for the first non-zero Steklov eigenvalue on star-shaped mean convex domains in hyperbolic space  $H^n$  for  $n \geq 4$ . In particular, when the domain is convex, our result gives an affirmative answer to Open Question 4.27 in [Colbois, Girouard, Gordon and Sher, Rev. Mat. Complut. 37 (2024)] for the hyperbolic space  $H^n$  when  $n \geq 4$ .

## **Parabolic Hermitian-Einstein metrics over nonKähler manifolds**

李希伦（北京大学）

Kobayashi-Hitchin correspondence demonstrates the equivalence between existence of canonical metrics on holomorphic bundles and slope stability. It has many extensions, and one of them is to consider metrics with singularities along some divisors, which is called parabolic bundles. The parabolic version of KH correspondence in Kähler setting was solved by Mehta, Seshadri, Li, Narasimhan and so on. In this talk, we generalize this to the nonKähler setting.

## **Existence of weighted cscK metrics**

刘亚雄（University of Maryland, USA）

In this talk, we study weighted cscK in the sense of Lahdili. We prove that on a smooth Kähler manifold, the G-coercivity of the weighted Mabuchi functional implies the existence of weighted-cscK (extremal) metric for the log-concave weight. On a projective klt variety, we prove that weighted uniform K-stability implies the G-corecivity of weighted Mabuchi functional. This is a joint work with Dr. Jiyan Han.

## **Sharp $L^\infty$ estimates for fully non-linear elliptic equations on compact complex manifolds**

乔钰翔（北京大学）

The  $L^\infty$  estimate plays an important role in the theory of fully non-linear elliptic equations on compact complex manifolds. It was a long-standing question whether Kołodziej's sharp  $L^\infty$  estimate could be proved by PDE methods instead of the pluripotential theory, until Guo-Phong-Tong gave an affirmative answer. Recently, we improved the result of Guo-Phong-Tong and proved that on a compact Kähler manifold, the oscillation of any admissible solution to a degenerate fully non-linear elliptic equation satisfying several structural conditions could be controlled by the  $L^1(\log L)^n(\log \log L)^r$  ( $r > n$ ) norm of the right-hand function (in a regularized form). For the case of compact Hermitian manifolds with non-degenerate background metrics, we proved a similar  $L^\infty$  estimate which improved that of Guo-Phong. An explicit example was constructed to show that these  $L^\infty$  estimates might fail when  $r \leq n-1$ . In this talk, we will discuss the results above.

## **Monge-Ampère type equations in two dimensions**

汪铃（北京大学）

In this talk, I will introduce the application of the partial Legendre transform to two-dimensional Monge-Ampère type equations, based on my recent joint works with Bin Zhou. In particular, I will demonstrate how the partial Legendre transform can be used to establish interior estimates and Liouville-type theorems for Monge-Ampère equations, linearized Monge-Ampère equations, as well as Monge-Ampère type fourth-order equations in two dimensions.

## **On the topology of the manifolds with positive intermediate curvature**

姚萱（Cornell University, USA）

We formulate a conjecture relating the topology of a manifold's universal cover with the existence of metrics with positive  $m$ -intermediate curvature. We prove the result for manifolds of dimension  $n \in \{3, 4, 5\}$  and for most choices of  $m$  when  $n=6$ . As a corollary, we show that a closed, aspherical 6-manifold cannot admit a metric with positive 4-intermediate curvature.