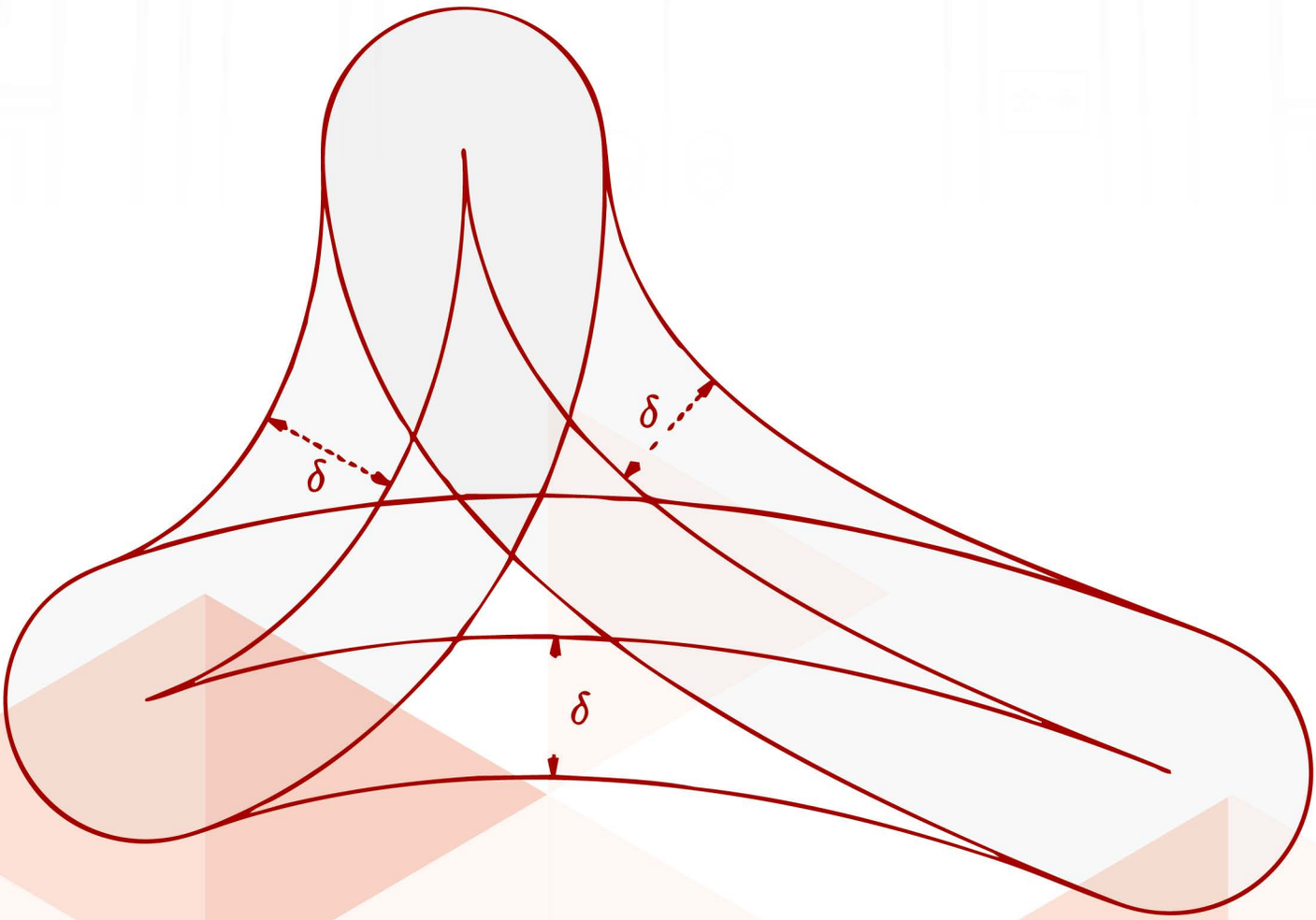
GEOMETRIC GROUP THEORY AND RELATED TOPICS

Peking University, Beijing, China

June 16-20, 2025



Supported by

Beijing International Center for Mathematical Research (BICMR)

List of Speakers

- Jason Behrstock (City University of New York)
- Inhyeok Choi (Connell University)
- Koji Fujiwara (Kyoto University)
- Victor Gerasimov (UFMG)
- Mark Hagen (University of Bristol)
- Jingyin Huang (Ohio State University)
- Vadim Kaimanovich (University of Ottawa)
- Ilya Kapovich (City University of New York)
- Andres Karlsson (Université de Genève)
- Sang-Hyun Kim (KIAS)
- Chris Leininger (Rice University)
- Marco Linton (ICMAT, Madrid)
- Vlad Markovic (Tsinghua university)
- Bruno Martelli (Università di Pisa)
- Damian Osajda (University of Copenhagen)
- Piotr Przytycki (McGill University)
- Kasra Rafi (University of Toronto)
- Alessandro Sisto (Heriot-Watt University)
- Ryokichi Tanaka (Kyoto University)
- Jing Tao (University of Oklahoma)
- Giulio Tiozzo (University of Toronto)

Conference venue:

Lecture Room, Jiayibing Building, Jingchunyuan 82, BICMR
(镜春园 82 号甲乙丙楼报告厅)

Live streaming:

VooV Meeting (腾讯会议)：398-2652-7885

Meeting Password: 250616

Organizing Committee

Yi Liu (Peking University)

Mahan Mj (Tata Institute of Fundamental Research)

Leonid Potyagailo (Université de Lille)

Wenyuan Yang (Peking University)

Conference Schedule

	Monday (16 June)	Tuesday (17 June)	Wednesday [1] (18 June)	Thursday (19 June)	Friday [2] (20 June)
09:00-09:30	Registration				
09:30-10:30	Vlad Markovic	Chris Leininger	Ilya Kapovich (online)	Vadim Kaimanovich	Koji Fujiwara
10:30-11:00	Group Photo	Tea break	Poster session	Tea break	
11:00-12:00	Kasra Rafi	Jing Tao	Marco Linton	Victor Gerasimov	Anders Karlsson
12:00-14:00	Lunch break				
14:00-15:00	Piotr Przytycki	Bruno Martelli	Free discussion	Jason Behrstock	Sanghyun Kim
15:00-15:30	Tea break			Tea break	Jingyin Huang
15:30-16:30	Alex Sisto	Damian Osjada		Giulio Tiozzo	Free discussion
16:40-17:40	Inhyeok Choi	Mark Hagen		Ryokichi Tanaka	
17:40 onwards	Free discussion				
18:30-20:00		Banquet			

Note:

1. Wednesday starts at 09:00, with a poster session 10:00-11:00.
   Zoom meeting ID: 976 6225 5234
   Password: 659193
2. The last talk on Friday afternoon starts at 15:10.

Contents

Title and Abstract

Random graphs and Coxeter groups

Jason Behrstock
City University of New York

Erdos and Renyi introduced a model for studying random graphs of a given density and proved that there is a sharp threshold at which lower density random graphs are disconnected and higher density ones are connected. This framework is extremely well-suited for studying right-angled Coxeter groups (RACGs) and we will describe some theorems analogous to the Erdos-Renyi in this context. Moreover, in trying to understand random groups, we've discovered some surprising new results, including for instance, a simple constraint on a presentation graph which forces the associated RACG to be relatively hyperbolic.

Genericity problem in groups with non-positive curvature

Inhyeok Choi
Connell University

Given a finitely generated group G and a property P concerning its elements, one can ask: Does a typical element of G satisfy property P? A natural interpretation of this question involves examining the asymptotic density of P-elements in word metric balls. In this talk, I will explain the current understanding of such statistical questions in group theory, with particular emphasis on relatively hyperbolic groups, mapping class groups (of surfaces) and $Out(F_n)$.

The Hardy-Littlewood maximal inequality for hyperbolic groups

Koji Fujiwara
Kyoto University

The Hardy-Littlewood maximal inequality is a fundamental inequality for a maximal operator for an L^1 functions on Euclidean spaces. Naor-Tao gave an interesting geometric proof of the Hardy-Littlewood maximal inequality for a free group. I explain how their argument extends to hyperbolic groups. This is a joint work with Amos Nevo.

A characterization of hyperbolic groups in terms of random walks

Victor Gerasimov
Universidade Federal de Minas Gerais

A probability measure μ on a group G is said to be *admissible* if:

1. it is symmetric: $\mu(g) = \mu(g^{-1})$;
2. the support $\mathrm{supp}_\mu := \{g \in G : \mu(g) \neq 0\}$ is finite;
3. the group G is generated by supp_μ.

Every admissible probability measure μ on G yields a Markov chain with the state set G, called *random walk* on G. We are looking for probabilistic properties of random walks on G that hold if and only if G is Gromov hyperbolic.

We are mainly interested in various versions of Ancona's property:

if the Gromov product with respect to the word metric is zero then the Gromov product with respect to the Green metric is uniformly bounded.

The probabilistic interpretation of Ancona's property: there exists $\epsilon > 0$ such that, for every word geodesic segment $[x, z]$ and every $y \in [x, z]$ the probability that a random path from x to z passes through y is greater than ϵ.

This is a joint work with Leonid Potyagailo.

Universal real cubings and asymptotic cones

Mark Hagen
University of Bristol

Around 2000, Dyubina-Polterovich gave an explicit construction showing that local data — in this case, the valence — determines a universal complete homogeneous real tree. Using work of Berestovskii-Plaut, for instance, one can make many free, transitive group actions on these trees. In another direction, the vertex set of the Cayley graph of a right-angled Artin group A is a median space that admits a free, transitive action and is reconstructable from "local data" — the presentation graph — in a somewhat similar way.

I will discuss a higher rank generalisation of universal real trees, namely universal real cubings, which are likewise built from local data and which admit a free, transitive action by a group that is uncountable but has RAAG-like features. The reason to consider such objects is that, if G is a suitable hierarchically hyperbolic group (for example, a mapping class group of a finite-type surface, or the fundamental group of a compact special cube complex), then all asymptotic cones of G turn out to be bilipschitz equivalent to universal real cubings. This result is one of two main steps in the proof that such G have unique (up to bilipschitz equivalence) asymptotic cones.

This talk is on joint work with Montserrat Casals-Ruiz and Ilya Kazachkov.

Exotic aspherical 4-manifolds

Jingyin Huang
Ohio State University

We show that there are closed, aspherical, smooth 4-manifolds that are homeomorphic but not diffeomorphic. This is joint work with Davis, Hayden, Ruberman, and Sunukjian.

Singularity of compound stationary measures

Vadim Kaimanovich
University of Ottawa

Any irreducible random walk on a hyperbolic group gives rise to the corresponding harmonic measure on the hyperbolic boundary. If two random walks have the same harmonic measure, then any compound random walk (whose step distribution is a convex combination or convolution of the original ones) also has the same harmonic measure. What happens if the original harmonic measures are just equivalent (rather than equal)? The talk is based on a joint work with Behrang Forghani.

On two-generator subgroups of mapping torus groups

Ilya Kapovich
City University of New York

Motivated by the results of Jaco and Shalen about 3-manifold groups, we prove that if F is a free group (of possibly infinite rank), $\phi : F \to F$ is an injective endomorphism of ϕ and $G_\phi = \langle F, t | txt^{-1} = \phi(x), x \in F \rangle$ is the mapping torus group of ϕ then every two-generator subgroup of G_ϕ is either free or a "finitary submapping torus." For a fully irreducible automorphism ϕ of a finite rank free group F_r this result implies that every two-generator subgroup of the free-by-cyclic group G_ϕ is either free, free abelian, a Klein bottle group or a subgroup of finite index in G_ϕ; and if $\phi \in Out(F_r)$ is fully irreducible and atoroidal then every two-generator subgroup of G_ϕ is either free or of finite index in G_ϕ.

This talk is based on joint paper with Naomi Andrew, Edgar A. Bering IV and Stefano Vidussi, with an appendix by Peter Shalen.

An alternative for Liouville groups acting on finite-dimensional CAT(0)-spaces

Anders Karlsson
Université de Genève

In joint work with H. Izeki, we establish the following alternative for finitely generated groups not admitting any non-constant bounded harmonic functions (Liouville groups), e.g. all groups of subexponential growth. Either every action on a finite-dimensional complete CAT(0)-space has a fixed point, or it admits an infinite finite-dimensional linear representation. The alternative implies that torsion groups or simple groups that are Liouville (e.g. groups of Grigorchuk, Nekrashevych and Matte Bon) cannot act without fixed point on finite-dimensional CAT(0) spaces. The proof uses a recent paper by Izeki on random walks, combined with ultralimits, equivariant harmonic maps, subharmonic functions and horofunctions.

First order rigidity of manifold homeomorphism groups

Sang-Hyun Kim
Korea Institute for Advanced Study

Two groups are elementarily equivalent if they have the same sets of true first order group theoretic sentences. We prove that if the homeomorphism groups of two compact topological manifolds are elementarily equivalent, then the manifolds are homeomorphic. This generalizes Whittaker's theorem on isomorphic homeomorphism groups (1963) without relying on it. For smooth manifolds, one can strengthen the result to obtain the existence of a diffeomorphism when the groups are the C^r diffeomorphism group and the C^s diffeomorphism group, respectively.

Joint work with Thomas Koberda (UVa) and Javier de la Nuez-Gonzalez (KIAS).

Atoroidal surface bundles

Chris Leininger
Rice University

I'll discuss my joint work with Autumn Kent on the first construction of atoroidal surface bundles over surfaces, or equivalently, purely pseudo-Anosov surface subgroups of the mapping class group. The main construction is a type-preserving homomorphism from the fundamental group of the figure-eight knot complement into the mapping class groups of a thrice punctured torus. Here, type preserving means that all loxodromic elements are mapped to pseudo-Anosov elements. Purely pseudo-Anosov surface subgroups come from restricting this homomorphism to any totally geodesic (or more generally, quasi-Fuchsian) surface subgroup, of which there are many.

Quasi-convex subgroups of free-by-cyclic groups

Marco Linton
ICMAT, Madrid

The well-known subgroup tameness theorem for hyperbolic 3-manifold groups characterises precisely when a finitely generated subgroup is quasi-convex. An immediate consequence is a characterisation of hyperbolic 3-manifold groups that are locally quasi-convex as those that do not contain compact surface-by-cyclic subgroups. Although a version of the subgroup tameness theorem for the class of free-by-cyclic groups remains a difficult open problem, I will instead show that an analogous characterisation of local quasi-convexity amongst free-by-cyclic groups does indeed hold. I will explain the ideas that go into the proof, discuss a generalisation to the relatively hyperbolic setting and mention some applications to one-relator groups.

TBA

Vlad Markovic
Tsinghua university

A 4-dimensional pseudo-anosov homeomorphism

Bruno Martelli
Università di Pisa

A celebrated theorem of Thurston states that a 3-manifold that fibers over the circle has a hyperbolic structure if and only if the monodromy is pseudo-Anosov. It is natural to wonder whether some version of this theorem holds in higher dimension. We will show that a natural strong version of this theorem holds at least in one case: there is a hyperbolic 5-manifold that fibers over the circle, and the monodromy is pseudo-Anosov in some very natural sense.

Drilling Hyperbolic Groups

Damian Osajda
University of Copenhagen

Drilling a closed hyperbolic 3-manifold along an embedded geodesic is a crucial technique in low-dimensional topology, transforming the fundamental group of the manifold into a relatively hyperbolic group. In this talk, we extend this concept by proving that, under appropriate conditions, a similar "drilling" operation can be applied to a (Gromov) hyperbolic group with the 2-sphere boundary.

Our primary motivations and applications revolve around the Cannon Conjecture, which states that if the Gromov boundary of a hyperbolic group is homeomorphic to the 2-sphere, then the group is virtually (i.e., up to a finite-index subgroup) the fundamental group of a closed 3-manifold of constant negative curvature. We also explore the relatively hyperbolic counterpart—the Toral Relative Cannon Conjecture.

Using drilling, we show that if the Toral Relative Cannon Conjecture holds, then the Cannon Conjecture is valid for all residually finite hyperbolic groups. The Toral Relative Cannon Conjecture appears more accessible, owing to the presence of additional structure—abelian parabolic subgroups.

This is joint work with Daniel Groves, Peter Haïssinsky, Jason Manning, Alessandro Sisto, and Genevieve Walsh.

A pair of Garside shadows

Piotr Przytycki
McGill University

This is joint work with Yeeka Yau. I will explain why minimal elements of cone type components and Shi components form "Garside shadows" in Coxeter groups. This leads to convenient path systems in their Cayley graphs.

What does a random surface look like?

Kasra Rafi
University of Toronto

Building on her seminal work regarding moduli space volumes for Riemann surfaces, Mirzakhani also calculated expected values for various geometric functions on moduli space. Notably, she examined the expected Cheeger constant, the injectivity radius at a random point, and the statistical distribution of different types of curves on surfaces of large genus. We will review several of Mirzakhani's key results, which collectively offer insights into the geometry of random surfaces in high genus. Following this, we will explore some extensions of her findings in the context of translation surfaces.

Asymptotically CAT(0) spaces and applications

Alessandro Sisto
Heriot-Watt University

The talk is based on joint work with Matt Durham and Yair Minsky where we construct metrics that satisfy a CAT(0)-like inequality involving a sublinear error for a large class of groups including mapping class groups and a lot of their quotients, most 3-manifold groups, extra-large Artin groups, extension of Veech groups and multicurve stabilisers, and many others. These metrics in turn allow one to construct contractible Rips complexes with "nice" compactifications, and to use those to show that the groups in our large class satisfy the Farrell-Jones conjecture.

Dimension conservation of harmonic measures in products of hyperbolic spaces

Ryokichi Tanaka
Kyoto University

We show that the harmonic measure on a product of boundaries satisfies dimension conservation for a random walk with non-elementary marginals on a countable group acting on a product of hyperbolic spaces under finite first moment condition.

Polygons and the shape of Thurston geodesics

Jing Tao
University of Oklahoma

In the 1980s Thurston introduced an asymmetric metric on the Teichmuller space of hyperbolic surfaces, measuring between each pair of surfaces the minimal Lipschitz constant needed to deform one into the other. Geodesics in this metric are highly non-unique, and, even with recent progress via harmonic-map limits, their explicit descriptions remain elusive. In this talk I will explore some basic problems about building Lipschitz maps between ideal polygons and show how they are related to the "shape" of geodesics in the Thurston metric.

The Poisson-Furstenberg boundary of discrete subgroups of semisimple Lie groups without moment conditions

Giulio Tiozzo

University of Toronto

The Poisson(-Furstenberg) boundary is a measure-theoretic object attached to a group equipped with a probability measure, and is closely related to the notion of harmonic function on the group. In many cases, the group is also endowed with a topological boundary arising from its geometric structure, and a recurring research theme is to identify the Poisson boundary with the topological boundary.

In this talk, we prove that the Poisson boundary of a random walk with finite entropy on a discrete subgroup of a semisimple Lie group can be identified with its Furstenberg boundary, without assuming any moment condition on the measure. Note that no pivoting theory will be needed.

Joint with K. Chawla, B. Forghani, and J. Frisch.

Hotel 1:

FX Hotel ZhongGuanCun (富驿时尚酒店中关村店).

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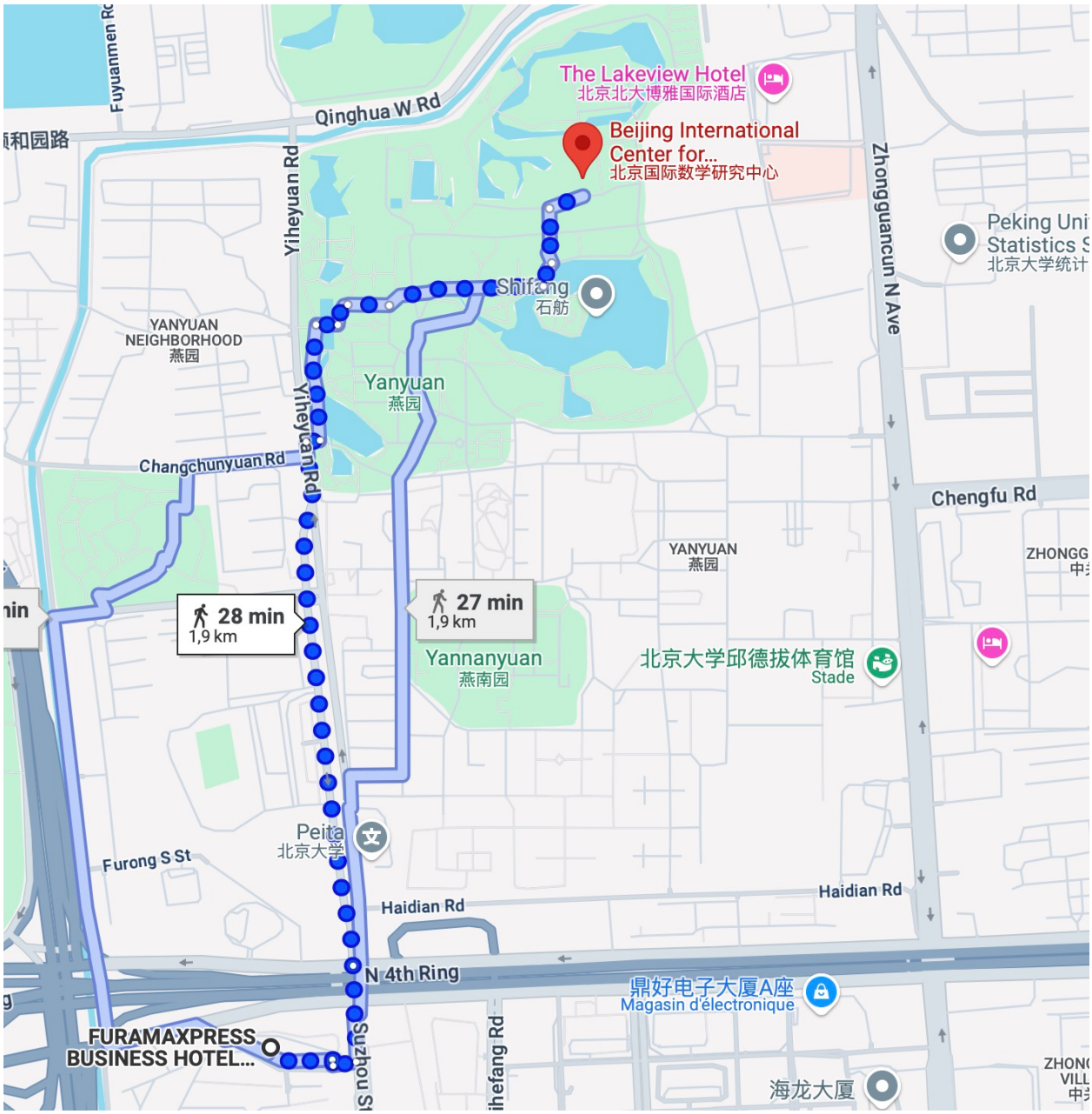

Hotel 2:

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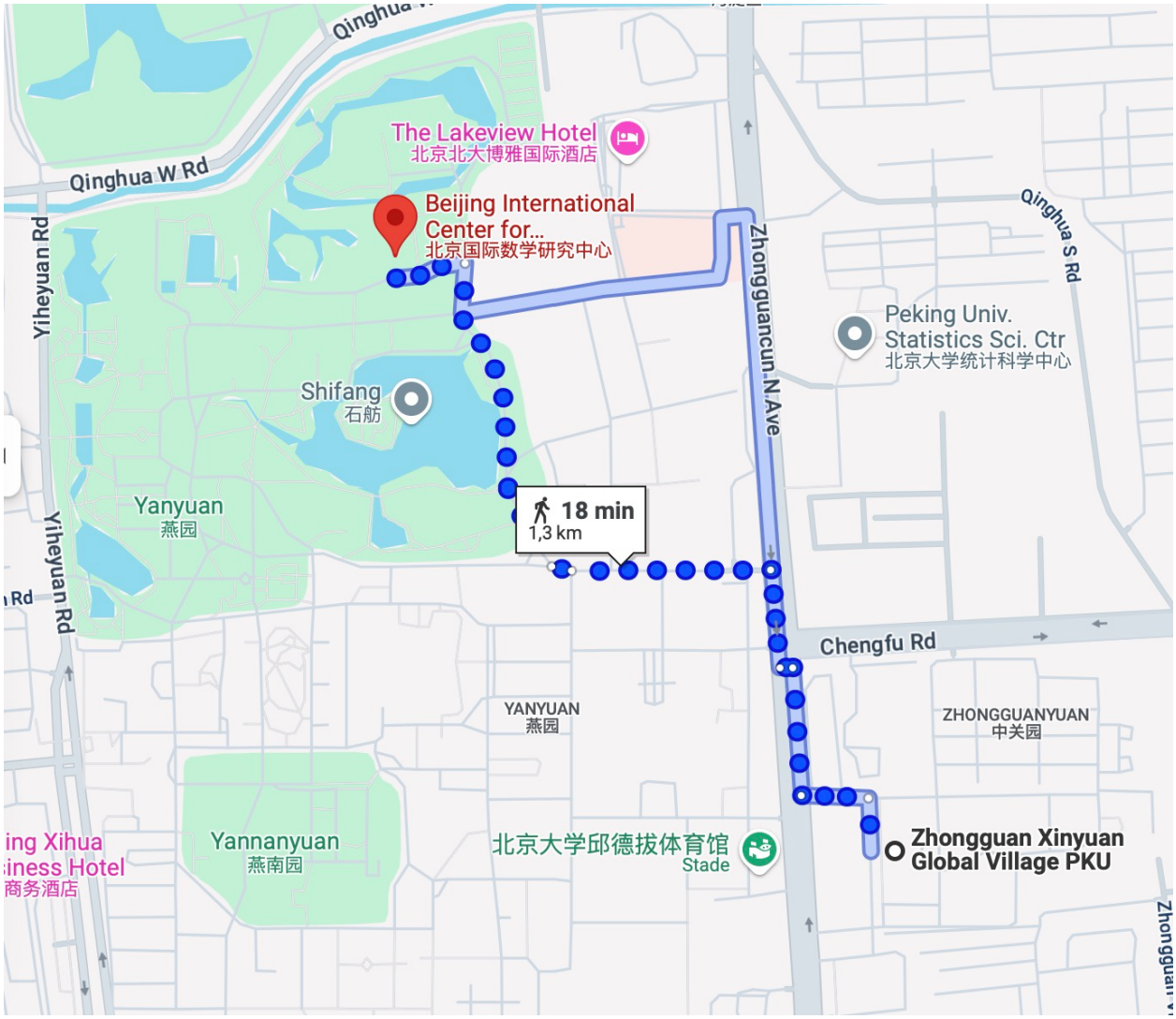


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