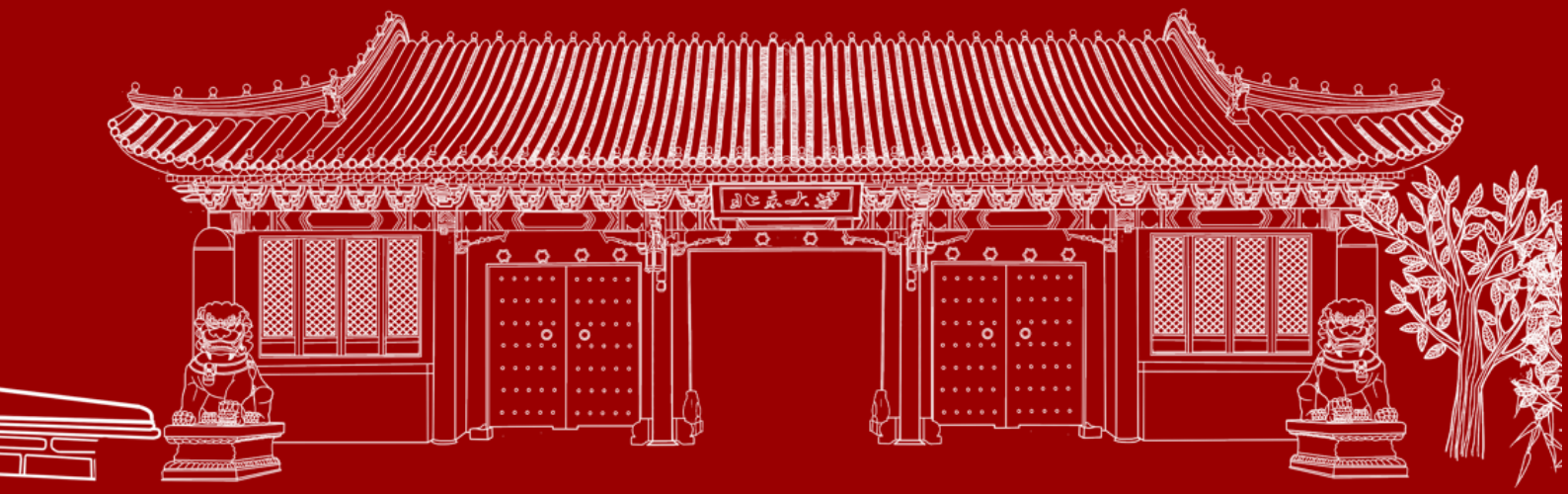
北京国际数学研究中心
BEIJING INTERNATIONAL CENTER FOR MATHEMATICAL RESEARCH

Conference on Holomorphic Dynamics and Fractal Geometry



Beijing International Center for Mathematical Research, PKU

13–18 January, 2025 · Beijing

Organizers

- Zhiqiang Li (Peking University, China)
- Junyi Xie (Peking University, China)

Speakers

- Walter Bergweiler — Christian-Albrechts-Universität zu Kiel, Germany
- Ilia Binder — University of Toronto, Canada
- Mario Bonk — University of California, Los Angeles, USA
- Danny Calegari — University of Chicago, USA
- Romain Dujardin — Sorbonne Université, France
- John Erik Fornaess — University of Michigan, Ann Arbor, USA; Norwegian University of Science and Technology, Norway
- Peter Haïssinsky — Aix-Marseille Université, France
- Hiroyuki Inou — Kyoto University, Japan
- Yunping Jiang — City University of New York, USA
- Luna Lomonaco — Instituto de Matemática Pura e Aplicada, Brazil
- Yusheng Luo — Cornell University, USA
- Mikhail Lyubich — Stony Brook University, USA
- Sergiy Merenkov — City University of New York, USA
- Daniel Meyer — University of Liverpool, UK
- Ngaiming Mok — Hong Kong University, China
- Sabyasachi Mukherjee — Tata Institute of Fundamental Research, India
- Pekka Pankka — University of Helsinki, Finland
- Kevin Pilgrim — Indiana University Bloomington, USA
- Dierk Schleicher — Aix-Marseille Université, France
- Weixiao Shen — Fudan University, China
- Mitsuhiro Shishikura — Kyoto University, Japan
- Tatiana Smirnova-Nagnibeda — University of Geneva, Switzerland

	Monday 13-Jan-25		Tuesday 14-Jan-25		Wednesday 15-Jan-25		Thursday 16-Jan-25		Friday 17-Jan-25		Saturday 18-Jan-25	
8:00 - 8:30	Registration											
	Speaker	**Chair**	**Speaker**	**Chair**	**Speaker**	**Chair**	**Speaker**	**Chair**	**Speaker**	**Chair**	**Speaker**	**Chair**
8:30 - 9:00	**Opening Address**: Gang Tian Lan Wen	Junyi Xie & Zhiqiang Li										
9:00 - 10:00	Mikhail Lyubich	Lan Wen	Weixiao Shen	Yuefei Wang	Ngaiming Mok	Guizhen Cui	Danny Calegari	Jinsong Liu	Mitsuhiro Shishikura	Wenxiang Sun	John Erik Fornaess	Aihua Fan
10:00 - 10:30	Tea		Tea		Tea		Tea		Tea		Tea	
10:30 - 11:30	Mario Bonk		Ilia Binder		Romain Dujardin		Dierk Schleicher		Walter Bergweiler		Yunping Jiang	
11:30 - 13:30	Lunch											
13:30 - 14:30	Pekka Pankka	Shaobo Gan	Tatiana Smirnova-Nagnibeda	Wenyuan Yang	Free Discussions		Peter Haïssinsky	Weiyuan Qiu	Luna Lomonaco	Yi Qi	Free Discussions	
14:30 - 15:00	Tea		Tea				Tea		Tea			
15:00 - 16:00	Daniel Meyer		Hiroyuki Inou				Kevin Pilgrim		Sabyasachi Mukherjee			
16:00 - 17:00	Sergiy Merenkov		Poster Session				Poster Session		Yusheng Luo			
17:00 onwards	Free Discussions		Conference Dinner				Free Discussions		Free Discussions			

ABSTRACT

Walter Bergweiler
Affiliation: Christian-Albrechts-Universität zu Kiel, Germany
Title: On the boundary of an immediate attracting basin of a hyperbolic entire function
Abstract: Let f be a transcendental entire function of finite order which has an attracting periodic point of period z_0 at least 2. Suppose that the set of singularities of the inverse of f is finite and contained in the component U of the Fatou set which contains z_0. Under an additional hypothesis we show that the intersection of ∂U with the escaping set of f has Hausdorff dimension 1. The additional hypothesis is satisfied for example if f has the form $f(z) = \int_0^z p(t)e^{q(t)}dt + c$ with polynomials p and q and a constant c. The result generalizes a theorem of Barański, Karpińska and Zdunik dealing with the case .
The hypothesis that all singularities of the inverse are contained in one component U of the Fatou set is essential: If this is not the case, then the intersection of ∂U with the escaping set may have Hausdorff dimension 2.
The results are joint work with Jie Ding.

Ilia Binder
Affiliation: University of Toronto, Canada
Title: Computing harmonic measure
Abstract: This talk explores the computability of one of the central objects of Potential Theory, the harmonic measure. It focuses on two key questions: what information about a domain is needed to compute its harmonic measure, and whether a single algorithm suffices for all points in the domain. The speaker will define the necessary concepts in Computability. The talk is based on collaboration with Mark Braverman, Adi Glucksam, Cristobal Rojas, and Michael Yampolsky.

Mario Bonk
Affiliation: University of California, Los Angeles, USA
Title: Conformal maps and critical points of Eisenstein series
Abstract: The location of the critical points of the Eisenstein series has recently gained some attention, in particular through the work of Zhijie Chen and Chang-Shou Lin. It is known that location of these critical points is the same as the location of the poles of some associated polymorphic functions. It seems not to have been observed that these polymorphic functions admit an explicit description as conformal maps between certain circular arc triangles. Based on this one can easily give a qualitative description of the location of the critical points of the lowest-weight Eisenstein series E_2, E_4, and E_6. In my talk I will give an introduction to this circle of ideas including classical themes about conformal mappings and the theory of modular forms.

Danny Calegari
Affiliation: University of Chicago, USA
Title: Combinatorics of the Tautological Lamination
Abstract: Laminations arise in complex dynamics by uniformizing the Fatou domain of polynomials. One dimensional slices of the parameter spaces of such polynomials are parameterized by "Tautological Laminations". We describe one such lamination that arises in the parameterization of the Cubic Shift Locus, and explain its unexpected connection to a problem in combinatorics.

Romain Dujardin
Affiliation: Sorbonne Université, France
Title: Some rigidity properties of polynomial automorphisms of $\mathbb{C}^2$
Abstract: In a recent work in collaboration with Serge Cantat, we address a number of classical rigidity properties from smooth and holomorphic dynamics in the context of polynomial automorphisms of $\mathbb{C}^2$. The primary motivation is a smooth conjugacy problem suggested by Friedland and Milnor in a celebrated paper, that we solve in the real-analytic case (under a non-reality assumption). Along the way we have to deal with other rigidity problems for a single map: smoothness of invariant foliations, smoothness of Julia sets. The properties of multipliers of saddle periodic orbits play an important role in these results. We also study some arithmetic questions related to these multipliers.

John Erik Fornaess
Affiliation: University of Michigan, Ann Arbor, USA; Norwegian University of Science and Technology, Norway
Title: Newtons method
Abstract: I report on recent joint work in progress with Hu, Truong and Watanabe Newtons method is used to find roots of complex polynomials in $\mathbb{C}$. Each root has a basin of attraction. Normally these basins have fractal boundaries. Truong has found a version of Newtons method where computer pictures indicate that the basins have smooth boundaries. We are working to see if this discovery can be proved rigorously.

Peter Haïssinsky
Affiliation: Aix-Marseille Université, France
Title: The geometry of potentials
Abstract: We will show how the thermodynamic formalism enables one to explore the conformal gauge associated to the dynamics of finite branched covers with suitable expanding behavior.

Hiroyuki Inou
Affiliation: Kyoto University, Japan
Title: Combinatorial rigidity of infinitely renormalizable cubic unicritical polynomials

Abstract: The local connectivity of the Mandelbrot set is equivalent to the combinatorial rigidity conjecture for the quadratic family. Henriksen gives a counterexample of cubic polynomials with infinitely many capture renormalizations.
In this talk, we construct a cubic polynomial with infinitely many cubic renormalizations. In particular, the infinitely renormalizable unicritical cubic polynomial with the same combinatorics is not combinatorially rigid in the space of all cubic polynomials.
We would also discuss its dynamical properties.

Yunping Jiang
Affiliation: The City University of New York-Graduate Center and Queens College
Title: Two rigidity results in one-dimensional dynamics
Abstract: Every smooth expanding circle endomorphism of degree $p > 1$ can be written as $F_{h,p} = h \circ f_p \circ h^{-1}$ where $f_p(x) = px \pmod 1$ and h is a circle homeomorphism normalized $h(0) = 0$. Let $\mathcal{C}^{1+H}$ be the space of all conjugacies h so that $F_{h,p}$ are $\mathcal{C}^{1+\alpha}$-expanding and define $\mathcal{SC}^{1+H} = \mathcal{C}^{1+H}/\text{Diff}$. From the foundational work of Krzyzewski and Szlenk, Ruelle, Sacksteder, Shub-Sullivan, etc., in the 1970s and 1980s, we know that each coset $\tau \in \mathcal{SC}^{1+H}$ contains one and only one h such that $F_{h,p}$ preserves the Lebesgue measure. In the 1980s, Sullivan observed that $\mathcal{C}^{1+H}$ is contained in the space of quasisymmetric circle homeomorphisms. Consequently, $\mathcal{SC}^{1+H}$ lies within the universal asymptotical Teichmüller curve $\mathcal{AT} \times \Delta$ as defined by Gardiner and Sullivan in 1990s. However, $\mathcal{SC}^{1+H}$ is not complete; Its completion, $\mathcal{US}$, consists of cosets of all quasisymmetric conjugacies h such that $F_{h,p}$ are uniformly symmetric modulo symmetric circle homeomorphisms. In a recent work (2022) with my students Adamski, Hu, and Wang, we established a symmetric rigidity result: If $F_{h_1,p}$ and $F_{h_2,p}$ are two circle endomorphisms preserving the Lebesgue measure with bounded geometry, and h is a symmetric conjugacy between them, then h must be the identity. This implies that each coset $\beta \in \mathcal{US}$ contains a unique $h \in \beta$ such that $F_{h,p}$ preserves the Lebesgue measure. Building on this, we recently addressed a question posed by Sullivan regarding rigidity under the p- and q-actions on the circle for a pair of coprime integers. Specifically, if h is a circle homeomorphism such that both $F_{h,p}$ and $F_{h,q}$ preserve the Lebesgue measure, and one of these actions satisfies a Lipschitz condition, is h necessarily the identity? We provide an affirmative answer to this question. Consequently, we reformulate the Furstenberg conjecture as a problem within the framework of quasiconformal mapping theory.

Luna Lomonaco
Affiliation: Instituto de Matemática Pura e Aplicada, Brazil
Title: The Mandelbrot set and its Satellite copies
Abstract: For a polynomial on the Riemann sphere, infinity is a (super) attracting fixed point, and the filled Julia set is the set of points with bounded orbit. Consider the quadratic family $P_c(z) = z^2 + c$. The Mandelbrot set M is the set of parameters c such that the filled Julia set of P_c is connected.
Computer experiments quickly reveal the existence of small homeomorphic copies of M inside itself; the existence of such copies was proved by Douady and Hubbard. Each little copy is either primitive (with a cusp on the boundary of its main cardioid region) or a

satellite (without a cusp). Lyubich proved that the primitive copies of M satisfy a stronger regularity condition: they are quasiconformally homeomorphic to M. The satellite copies are not quasiconformally homeomorphic to M (as we cannot straighten a cusp quasiconformally), but are they mutually quasiconformally homeomorphic? In joint work with C. Petersen we prove that the answer is negative in general, but positive in the case the satellite copies have rotation numbers with the same denominator.

Yusheng Luo
Affiliation: Cornell University, USA
Title: Uniformization of gasket Julia set
Abstract: In this talk, we present necessary and sufficient conditions for a gasket Julia set to be uniformized by a round gasket via a quasiconformal or David homeomorphism of the sphere. Previously, such results were known only for the Sierpiński carpet Julia set, due to the work of Bonk-Lyubich-Merenkov. The presence of touching points of the complementary components makes analysis subtler. We restrict to rational maps without Julia critical points. Under this condition, we show that a Julia set can be quasiconformally uniformized by a round gasket if and only if it is a fat gasket. We also prove that a Julia set can be uniformized by a round gasket with a David homeomorphism if and only if every Fatou component is a quasidisk. Our theorem applies to show that gasket Julia sets and limit sets can be locally quasiconformally homeomorphic, revealing a different behavior compared to the Sierpiński carpet setting. This is a joint work with Dimitrios Ntalampekos.

Mikhail Lyubich
Affiliation: Stony Brook University, USA
Title: Uniform sectorial bounds over the Main cardioid
Abstract: Sectorial bounds give a geometric control for the rotational dynamics in all scales. They originated in the work of Yoccoz concerning dynamics of neutral germs and in the work of Ecalle and Voronin concerning parabolic dynamics. About 20 years ago Inou and Shishikura proved remarkable uniform sectorial bounds for near parabolic quadrtaic polynomials, and this result found many deep applications. By totally different methods, such bounds have recently been established for all rotation numbers (in a joint work with Dima Dudko), which opens an opportunity for a complete semi-global understanding of neutral quadratic polynomials. We will give an overview of these developments.

Sergiy Merenkov
Affiliation: City University of New York, USA
Title: Local rigidity in dynamics
Abstract: I will discuss local quasisymmetric rigidity results for dynamical Sierpiński carpets. For example, in principle one can locally quasisymmetrically distinguish a Sierpiński carpet Julia set of a hyperbolic rational map from the limit set of a convex cocompact Kleinian group, which is not true for gasket-like Julia and limit sets. Such carpet rigidity

results follow from uniqueness properties of so-called Schottky maps between relative Schottky sets, developed about a decade ago. Relative Schottky sets are residual sets obtained by removing from planar domains collections of open geometric disks with disjoint closures. Schottky maps are local quasisymmetries between Schottky sets, that turn out to possess a certain conformality property.

Daniel Meyer
Affiliation: University of Liverpool, UK
Title: Extension of Folding maps
Abstract: A folding map is given by a piece of paper (i.e., an k-gon) that is folded and mapped to itself. Such a map may be obtained from a real rational map where $z \in \mathbb{C}$ and its complex conjugate $\bar{z}$ are identified. Using natural identifications, this is turn yields a generalized pseudo-Anosov map, which will be an invertible map on the 3-sphere. This is joint work with Andre de Carvalho and Toby Hall.

Ngaiming Mok
Affiliation: Hong Kong University, China
Title: Holomorphic dynamics in relation to rigidity phenomena on bounded symmetric domains and their quotient spaces
Abstract: We will discuss some of the works of the speaker's involving results or methods of holomorphic dynamics in relation to rigidity phenomena on bounded symmetric domains Ω, their quotient spaces and quotient spaces of their algebraic subvarieties, covering (1) characterization of commutants of Hecke correspondences on irreducible bounded symmetric domains Ω and (2) functional transcendence concerning $X_\Gamma := \Omega/\Gamma$ for arbitrary cocompac lattices Γ, and (3) rigidity of Γ-equivariant holomorphic maps from a bounded symmetric domain Ω of rank ≥ 2 into bounded domains on Stein manifolds for irreducible lattices $\Gamma \subset \mathrm{Aut}(\Omega)$.
For (1) we recall the reduction of a problem of Clozel-Ullmo (2003) concerning the characterization of commutants of Hecke correspondences which reduces to a conjecture characterizing measure-preserving germs of holomorphic maps on Ω, and the solution of Mok-Ng (*J. Reine Angew. Math.*, 2012) using results in CR geometry due to Webster and Huang. For (2) we will discuss a uniformization theorem of Chan-Mok (*J. Diff. Geom.*, 2022) on projective varieties uniformized by algebraic subvarieties of bounded symmetric domains and furthermore a recent proof of the speaker's on the analogue of the Ax-Lindemann theorem on $X_\Gamma := \Omega/\Gamma$ for *arbitrary* cocompact lattices $\Gamma \subset \mathrm{Aut}(\Omega)$, both relying on the rescaling argument on algebraic subvarieties $Z \subset \Omega$ related to holomorphic isometric embeddings of the Poincaré disk or higher-dimensional complex unit balls $\mathbb{B}^m$. For (3) we will discuss a result of Mok-Wong (to appear in *Algebraic Geometry and Physics*, 2025) proving that, for an irreducible torsion-free lattice $\Gamma \subset \mathrm{Aut}(\Omega)$ of an arbitrary bounded symmetric domain Ω and writing $X_\Gamma := \Omega/\Gamma$, a Γ-equivariant holomorphic map $F : \Omega \to D$ into a bounded domain D on a Stein manifold is necessarily a biholomorphism provided that F is a lifting of $f : X_\Gamma \to D/\Gamma' =: Y_{\Gamma'}$ inducing an isomorphism $f_* : \Gamma \xrightarrow{\cong} \Gamma'$ on fundamental groups and $Y_{\Gamma'}$ is of finite volume with respect to the Kobayashi-Royden volume form. A crucial part of the proof involves a study using

Moore's ergodicity theorem on semisimple Lie groups on the pull-back $F^*(H^\infty(D))$ by F of the complex vector space $H^\infty(D)$ of bounded holomorphic functions on D, and the rigidity theorem may be regarded as an application of this study to Kähler geometry on the hull of holomorphy $\widehat{D}$ of D with respect to the canonical complete Kähler-Einstein metric.

Sabyasachi Mukherjee
Affiliation: Tata Institute of Fundamental Research, India
Title: Where connectedness loci meet Teichmüller spaces
Abstract: Various connections and philosophical analogies exist between two branches of conformal dynamics; namely, rational dynamics on the Riemann sphere and actions of Kleinian groups. In an attempt to study these common features in a unified framework, we construct a space of combinations/matings of complex polynomials and Fuchsian groups. These matings live on the boundary of the space of polynomial-like maps, and can also be viewed as algebraic correspondences. This gives rise to "products" of Teichmüller spaces of genus zero orbifolds and connectedness loci of polynomials inside the space of algebraic correspondences. We will discuss compactifications of such copies of Teichmüller spaces, and end with a host of open questions.

Pekka Pankka
Affiliation: University of Helsinki, Finland
Title: Visual spheres of metrically expanding uniformly quasiregular maps
Abstract: Uniformly quasiregular (UQR) mappings were introduced in the 1990's by Iwaniec and Martin as a higher dimensional (quasi)conformal counterpart for holomorphic dynamics. Here uniformity refers to the property that the distortion of the self-map stays uniformly bounded under iteration. Such mappings carry invariant measurable conformal structure, and the map is a (non-smooth) conformal mapping with respect to this structure. In this talk, I will discuss a quasisymmetric uniformization problem for visual metrics of expanding maps on topological spheres and its connection to UQR mappings and to their metric variant, uniform branched quasisymmetries. This is joint work with Zhiqiang Li (PKU) and Hanyun Zheng (PKU).

Kevin Pilgrim
Affiliation: Indiana University Bloomington, USA
Title: Dynamics on moduli spaces of rational maps
Abstract: Via lifting of complex structures, a branched covering $f : (S^2, P) \to (S^2, P)$ with finite post-critical set P induces an algebraic correspondence on the moduli space of conformal structures on $S^2 \setminus P$. We show that the limit space of this correspondence is compact if and only if the mapping class biset of considered by Bartholdi-Dudko is contracting over the pure mapping class group; equivalently, this correspondence is uniformly hyperbolic near its limit space. I will also discuss various notions of non-uniform hyperbolicity for such correspondences, and some natural open problems. This is joint ongoing work with Walter Parry.

Dierk Schleicher
Affiliation: Aix-Marseille Université, France
Title: The Weierstrass and Ehrlich-Aberth root finders as dynamical systems
Abstract: Finding roots of complex polynomials in one variable is a fundamental challenge at the interface of theoretical and applied mathematics, as well as in many applications. Many methods have been proposed; the best known is Newton's method that tries to find a single root at a time. There are several methods that try to approximate all roots at the same time. The best known are the Weierstrass—Durand—Kerner and Ehrlich—Aberth methods. These are known to work very well in practice, but little is known about their properties as global dynamical systems. We discuss various global dynamical properties, especially the search for attracting periodic orbits (which rule out general convergence) and orbits that diverge to infinity. This reports on joint work, partially still in progress, with Bernhard Reinke, Michael Stoll, and others.

Weixiao Shen
Affiliation: Fudan University, China
Title: No wandering interval for asymmetric unimidal map
Abstract: I will discuss a joint work with Jorge Olivares, where we prove that an asymmetric unimodal map (i.e. with different left and right critical order) has no wandering interval.

Mitsuhiro Shishikura
Affiliation: Kyoto University, Japan
Title: Trees for Thurston's theorem for topological branched coverings of 2-sphere
Abstract: Thurston's theorem characterizes postcriticallt finite self-branched coverings of 2 sphere which are equivalent to rational maps. The key of his theory was to reduce the problem to an iteration in the Teichmüller space. When a map is not equivalent to a rational map, the iteration tends to the boundary of the Teichmüller space, and this is characterized by the Thurston obstruction, which is a multi-curve in the complement of the postcritical set with certain property. However, checking this criterion is not always easy, because one has to check the condition for infinitely many possible multi-curves. When an obstruction is given, we introduce a tree with a piecewise linear map. Using these objects in some cases, we show the non-existence of obstruction, i.e. the realizability as a rational map.

Tatiana Smirnova-Nagnibeda
Affiliation: University of Geneva, Switzerland
Title: Self-similar groups and spectra of graphs
Abstract: Self-similar groups are defined by their actions on infinite regular rooted trees and give rise to interesting families of finite and infinite graphs exhibiting self-similar features. In spectral graph theory, these provide interesting examples such as co-spectral graphs, graphs admitting exotic types of spectra and spectral measure, etc. Their self-similar nature allows to various techniques to study them, from approximation by finite graphs to renormalization.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