北京国际数学研究中心
BEIJING INTERNATIONAL CENTER FOR MATHEMATICAL RESEARCH

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Mathematical Finance and Insurance

Beijing International Center for Mathematical Research
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Mini-courses

BSDEs and G-Expectations in Finance

Shige Peng (Shandong University)

How to deal with probability model uncertainties in finance is an important and challenging problem. In this lecture we give an introduction to the theory of backward stochastic different equations (BSDE) and nonlinear G-expectations., which are powerful tools in financial pricing and risk measuring.

Totally Inaccessible Stopping Times; Applications to Credit Risk

Philip Protter (Columbia University)

In a dynamic setting random times model when random events happen. In the 1950s and 1960s, it became clear the useful random times were those known as stopping times. Stopping times in turn were classified as three types: predictable, accessible, and totally inaccessible. These lectures will focus on those times known as totally inaccessible times. Such times arise in nature as jump times of Markov processes.

Assuming T is a stopping time, the process $1_{T\geq t}$ is adapted and has non-decreasing paths, whence it is a submartingale. The Doob-Meyer decomposition theorem tells us that $1_{T\geq t}$ has a unique predictable compensator A with $A_0 = 0$ and nondecreasing paths, such that $1_{T\geq t} - A_t = M_t$ is a martingale. When T is a predictable time the process A equals $1_{T\geq t}$, and the martingale M is identically zero. When T is totally inaccessible the process A is non-trivial and is known as the compensator. Since T is often known as a time when something bad happens, A has an interpretation as a hazard rate.

Often – but not always – A has the form

$A_t = \int_0^{t\wedge T} \lambda_s ds$, where λ is a nonnegative, adapted process. (1)

In applications (including in Credit Risk Theory) one estimates the process $(\lambda_s)_{s\geq 0}$ to infer knowledge of A and therefore of T. Given an A, then a standard construction of T is created using what is known as a Cox construction. If in contrast one begins with T, one can ask what form the process A takes. For example, does it resemble the A of (1)? We give sufficient conditions for it to do so.

How this works and its implications will be the primary focus of this course.

Invited Talks

Financial market models with marked thin information flow

Anna Aksamit (University of Sydney)

We propose a model with disclosure of nontrivial information at a future date. Such model covers situations of a central bank announcing interest rate decisions at one of the bank meeting dates or a company revealing decision of management, which are vital in making investment choices. Abstract informational model consists of a random time which is built of stopping times from a reference filtration and random variable satisfying a generalised Jacod's hypothesis. In this setup we focus on establishing fundamental properties of a model including arbitrage opportunities and market completeness. We also provide further financial applications and insights. This is a joint work with Claudio Fontana.

Sensitivity Estimates with Computable Bias Bounds

Ning Cai (Hong Kong University of Science and Technology, Guangzhou)

The likelihood ratio method (LRM) is widely used to estimate sensitivities in risk management. Constructions of the LRM estimators depend heavily on the computations of probability density functions (and their derivatives) of the underlying models, which are usually known only through their Laplace transforms under many popular financial models. We propose a Laplace inversion based LRM with computable bias bounds under these models. By selecting the algorithm parameters appropriately, we can obtain LRM estimators with any desired bias level. In addition, some asymptotic properties of our LRM estimators are also investigated. Numerical experiments indicate that our method performs well under a broad range of popular financial models.

A Two Time-Scale Evolutionary Game Approach to Multi-Agent Reinforcement Learning and its Application in Algorithmic Collusion

Nan Chen (Chinese University of Hong Kong)

We propose a two time-scale evolutionary game approach to solving general-sum multiagent reinforcement learning (MARL) problems in this paper. Different from the existent literature that requires to solve Nash equilibrium strategies, exactly or approximately, in each episode of learning, the new approach incorporates two innovative designs. First, we propose a simple perturbed best response based protocol to update policies. This enables us to avoid the computationally expensive step of finding the exact equilibrium at each state. Second, we update agents' policies and their Q-values in two different time scales to overcome the non-stationary obstacle in learning process. The new approach provably converges to approximate Nash equilibria of MARL problems without imposing the global optima or saddle point conditions, two restrictive assumptions that are typically needed in the literature. This framework can be easily extended to the setups of unobservable policies and private states.

AI-powered algorithms are now widely adopted in marketplaces to price goods and services. However, serious concerns have been raised by the regulators and academia about the possibility that these algorithms may learn to collude through their strategic interactions. Researchers predominately use Q-learning to model the behavior of pricing algorithms, which is short of convergence guarantees. Our approach provides a novel framework for algorithmic collusion studies. This is a joint work with Ruixun Zhang and Yumin Xu (Peking University) and Mingyue Zhong (Tsinghua University).

Uncovering Market Disorder and Liquidity Trends Detection

Etienne Chevalier (University Evry)

The primary objective of this paper is to conceive and develop a new methodology to detect notable changes in liquidity within an order-driven market. We study a market liquidity model which allows us to dynamically quantify the level of liquidity of a traded asset using its limit order book data. The proposed metric holds potential for enhancing the aggressiveness of optimal execution algorithms, minimizing market impact and transaction costs, and serving as a reliable indicator of market liquidity for market makers. As part of our approach, we employ Marked Hawkes processes to model trades-through which constitute our liquidity proxy. Subsequently, our focus lies in accurately identifying the moment when a significant increase or decrease in its intensity takes place. We consider the minimax quickest detection problem of unobservable changes in the intensity of a doubly-stochastic Poisson process. The goal is to develop a stopping rule that minimizes the robust Lorden criterion, measured in terms of the number of events until detection, for both worst-case delay and false alarm constraint. We prove our procedure's optimality in the case of a Cox process with simultaneous jumps, while considering a finite time horizon. Finally, this novel approach is empirically validated by means of real market data analyses. This is a joint work with Yadh Hafsi and Vathana Ly Vath.

Graph clustering and ranking for statistical arbitrage and lead-lag detection in equity markets

Mihai Cucuringu (Oxford University)

Abstract: We develop spectral methods for clustering heterogeneous networks, in the setting of signed and directed networks, and demonstrate their benefits on networks arising from stochastic block models and financial multivariate time series data, where one is often interested in clustering assets that exhibit similar contemporaneous behavior. We propose a framework to construct statistical arbitrage portfolios with graph clustering algorithms. Another task of interest is that of uncovering lead-lag relationships in high-dimensional multivariate time series. In such settings, certain groups of variables partially lead the evolution of the system, while other variables follow this evolution with a time delay, resulting in a lead-lag structure, which, at the pairwise level, can be encoded as edges of a directed network. Detecting clusters which exhibit a certain notion of pairwise flow imbalance amounts to identifying baskets of assets which lead-lag each other. We leverage graph clustering and ranking algorithms for the task of lead-lag detection, and demonstrate that our proposed methodology is able to detect statistically significant lead-lag clusters in the US equity market, and test their robustness on synthetically generated lagged factor models. We study the composition of the uncovered lead-lag equity clusters, compare performance at different time frequencies and against established approaches from the lead-lag literature for portfolio construction.

Learning Optimal Investment Strategy with Transaction Costs via a Randomized Dynkin Game

Min Dai (Hong Kong Polytechnic University)

Abstract: We develop a reinforcement learning method to learn optimal investment strategy in the presence of transaction costs. Using a connection between singular control and a Dynkin game for portfolio choice with transaction costs, we design a reinforcement learning algorithm to learn the optimal strategy through a related randomized Dynkin game, where a regularization term is incorporated to encourage exploration. Numerical results are presented to demonstrate our algorithm.

On the Expansion of Risk Pooling

Runhuan Feng (Tsinghua University)

Risk pooling has been an increasingly critical tool for managing catastrophe risks between corporations and among nation states with examples including multinational pooling and catastrophe risk pooling. While there has been a rich literature on its practices, little is known from a theoretical viewpoint operational strategies of risk pools. For example, what is the optimal size of a risk pool through expansion? How should members of a risk pool accept new members? The paper offers the first of its kind to study the stability, different notions of consensus for the expansion of risk pools and the impact of reinsurance on a risk pool under various assumptions of economic fairness. It reveals that, in a ballot-box-voting setting where each member votes to accept a candidate, the risk pool tends to be conservative and prefers candidates of low risks. In contrast, in a voting-by-feet setting where a candidate is always allowed to join but any existing member can leave, the risk pool becomes risk seeking and grows with candidates of high risks. The impact of a reinsurance on tail risk has mixed effects on the expansion of risk pools.

Optimal Dynamic propagation of Convexity, for Utility optimization and Convex Pricing

Nicole El Karoui (Sorbonne University)

In economics or in finance, decision making focuses on the selection of optimal choices based on expected utility-(concave) criterium $\{U(t, z)\}$. But, the optimal choice $\{X_t(x)\}$ is more easier accessible to the decision-maker, as in e-commerce. Following, Samuelson (1933) the practical problem is forward, that is to recover a stochastic dynamic utility process $\{U(t,z)\}$ from the observed(optimal) process $\{X_t(x)\}$. As usual in convex analysis, with the pair (X,U), we associate a pair (Y,V) where $\{Y(t,y)\}$ is a state price density process orthogonal to X : $\{X_t(x)Y_t(y)\}$ are supermartingale; V (t, y) is the Fenchel convex conjugate $V(t, y) = \sup_x\{xy - u(x)\}$, $(x > 0, y > 0)$. The processes $\{U(t,X_t(x))\}$, $\{V(t,Y_t(y))\}$, $\{X_t(x)Y_t(u_X(x))\}$ are assumed martingales because of optimality. $X_t(x)$ is just assumed increasing in x, and then invertible in x. Then, a forward "revealed" utility is obtain by a pathwise method from the first order condition, $U_Z(t,X_t(x)) = Y_t(u_Z(x))$, when $X_t(x)$ is assumed increasing in x. Moreover, any processes Z_t such that $\{Z_tY_t(y)\}$ is a supermartingale for any y, is sub-optimal in the sense that $\{U(t,Z_t)\}$ is a supermartingale.

In the derivatives market, standard European contracts have convex pay-offs of a underlying $\{S_t(x)\}$; the convexity also concerns the stochastic pricing kernel $\Phi(t, z)$ whose the value at $(t, S_t(x))$ is the price of contract $\Phi(T , S_T(x))$ in a backward framework. In the Black-Scholes model where $S_t(x)$ is linear in x, the pricing kernel $\Phi(t, z)$ is deterministic and convex in z; the same property holds true when $S_t(x)$ has a stochastic regular local volatility, as shown in the paper on the Robustness of BS Formula (1992-98). The previous forward method can be used to generalize the framework to more general situation. But, practitioners are also concerned by the hedging portfolio of the contract, and the convexity has been a key property in the martingale market for simplicity, to obtain superhedging strategy when the the volatility is augmented by error for example. we obtain, in our general context, the same result using Ito's Ventzel formula, but we are looking for a more direct proof (work in progress). Joint work with Mrad M. and Hillairet. C.

Design Liquidity Pools on Automated Market Makers

Xuedong He (Chinese University of Hong Kong)

Automated market makers are a popular type of decentralized exchanges in which users trade assets with each other directly and automat- ically through a liquidity pool and a fixed pricing function. The liquidity provider contributes to the liquidity pool by supplying assets to the pool and in return they earn transaction fees from traders who trade through the pool. We propose a model of optimal liquidity provision in which the risk-averse liquidity provider decides the investment proportion of wealth she would like to supply to the pool, trade in an centralized market, and consume in multiple periods. We derive the liquidity provider's optimal strategy by dynamic programming and numerically find the optimal liquidity pool that maximizes the liquidity provider's utility. Our findings indicate that the exchange rate volatility on the centralized market exerts a positive effect on optimal transaction fee. Moreover, the optimal constant mean pricing formula is found to be related to the relative performance of the underlying assets on the centralized market. This is a joint work with Chen Yang and Yutian Zhou.

Shrinkage of semimartingales

Monique Jeanblanc (University Evry)

We consider a semimartingale in some filtration, and our aim is to deduce, under adequate hypotheses the semimartingale decomposition of its projection on a smaller filtration. This can be applied to BSDEs in two filtrations.

The derivative of killed processes

Arturo Kohatsu-Higa (Ritsumeikan University)

In various fields including some in finance it is useful to have the concept of derivative of a killed process or a stopping time. We have started a project where such concepts can be discussed. Presently, we are developing some theoretical results that can be later applied. We will give a brief description of the project, achieved goals and some heuristics. I will present some results that have been obtained in joint work with Dan Crisan (Imperial College) and Fabio Antonelli (University of L'Aquila).

A new approach in two-dimensional heavy-tailed distributions.

Dimitrios G. Konstantinides (University of Aegean)

We consider a new approach in the definition of two-dimensional heavy-tailed distributions. Namely, we introduce the classes of two-dimensional long-tailed, of two- dimensional dominatedly varying and of two-dimensional consistently varying distributions. Next, we define the closure property with respect to two-dimensional convolution and to joint max-sum equivalence in order to study if they are satisfied by these classes. Fur- ther we examine the joint behavior of two random sums, under generalized tail asymptotic independence. Afterward we study the closure property under scalar product and two-dimensional product convolution and by these results we extended our main result in the case of jointly randomly weighted sums. Our results contained some applications

where we establish the asymptotic expression of the ruin probability in a two-dimensional discrete-time risk model. Joint work with Charalampos D. Passalidis.

Linearisation techniques and the dual algorithm for a class of mixed singular/continuous control problems in reinsurance

Juan Li (Shandong University)

We focus on linearisation techniques for a class of mixed singular/continuous control problems and ensuing algorithms. The motivation comes from (re)insurance prob- lems with reserve-dependent premiums with Cramér-Lundberg claims, by allowing singular dividend payments and capital injections. Using variational techniques and embedding the trajectories in an appropriate family of occupation measures, we provide the linearisation of such problems in which the continuous control is given by reinsurance policies and the singular one by dividends and capital injections. The linearisation translates into a dual dynamic programming (DDP) algorithm. An important part of the paper is dedicated to structural considerations allowing reasonable implementation. We also hint connections to methods relying on moment sum of squares and LMI (linear matrix inequality)-relaxations to approximate the optimal candidates.
Based on joint works with Dan Goreac (Shandong University) and Boxiang Xu (Shandong University).

Bermudan option: A entropy regularization approach.

Libo Li (University of New South Wales)

The goal of this work is to examine the numerical pricing of Bermudan option in the context of reinforcement learning through introducing entropy regularization/penalization. Our approach is based on BSDEs and we show that as the penalization parameter goes to zero the entropy regularized BSDE will converge to the theoretical price of the Bermudan option.

Continuous-state branching processes in varying environments

Zenghu Li (Beijing Normal University)

Abstract. Continuous-state branching processes without or with immigration in varying or random environments arise naturally in a number of applications. We give a characterization of the (inhomogeneous) cumulant semigroup of the process in varying environments in terms of a reverse evolution family of Bernstein functions. This is the basis of the study of processes in random environments.

(Rough) continuous states branching processes and their applications in volatility modelling

Simone Scotti (University of Pise)

This presentation is based on three different papers sharing the idea to joint calibrate SPX and VIX implied volatilities and many empirical facts in a parsimonious way. The three models are exponential affine and then the Fourier-Laplace transform of the log returns and the square of the volatility index can be computed explicitly in terms of solutions of deterministic pseudo-Riccati (Volterra) equation.
The first model is driven by a branching alpha-stable and then also reproduce the empirical results on jump infinite activity by Todorov and Tauchen.

The two other models are based on extension of Hawkes processes since the intensity of the jumps coincides with the volatility process itself. In the last case, the volatility is rough.
We show that our parsimonious setup is able to simultaneously capture, with a high precision, the behavior of the implied volatility smile for both S&P 500 and VIX options.
Besides the applications in mathematical finance many open questions arises that can resumed by which properties of CSBPI are preserved when a kernel is added?

A representation of stochastic processes and its application in the exit contract problems

Xiaolu Tan (Chinese University of Hong Kong)

We first recall Bank-El Karoui's representation theorem for stochastic processes and then provide a mean-field extension of the theorem. We next study an exit contract optimization problem, where a principal provides a universal exit contract to (finitely or infinitely) many heterogeneous agents. With the same contract, the agents may have different optimal exit times. The problem consists in finding the optimal universal contract from the principal's point of view. Under some structural conditions, we show how the exit contract problem can be solved by Bank-El Karoui's representation theorem and its extensions.

Modeling and Pricing Credit Risk with a Focus on Recovery Risk

Qihe Tang (University of New South Wales)

Abstract: Consider a defaultable bond traded in the financial market that is vulnerable to shocks. Its recovery payment comprises both an endogenous component contingent on the market performance until default and an exogenous component to account for its unpredictability. The latter poses some technical challenges to the pricing task. Assuming that the state variables for the default intensity, interest rate, and reference rate jointly follow a jump-diffusion process with drift and volatility coefficients governed by the market's regime and with jumps induced by the shocks, we construct a risk-neutral pricing measure that prices all risk sources in an integrated manner. In doing so, we disentangle the bond's risk sources into five independent types of risks. The developed pricing framework is applicable to most defaultable bonds and credit derivatives. Our work captures three important features: exogenous shocks that impact every aspect of the market, regime shifts of the market, and unpredictability embedded in the recovery. This talk is based on a joint work with Haibo Liu (Purdue University).

Asymptotic Subadditivity/Superadditivity of Value-at-Risk under Tail Dependence

Jingping Yang (Peking University)

We consider the asymptotic subadditivity and superadditivity properties of Value-at-Risk for multiple risks whose copula admits a stable tail dependence function (STDF). For the purpose, a marginal region is defined by the marginal distributions of the multiple risks, and a stochastic order named tail concave order is presented for comparing individual tail risks. We prove that asymptotic subadditivity of VaR holds when individual risks are smaller than regularly varying random variables with index -1 under the tail concave order. We also provide sufficient conditions for VaR being asymptotically superadditive. For two multiple risks sharing the same copula function and satisfying the tail concave order, a comparison result on the asymptotic subadditivity/superadditivity of VaR is given. Asymptotic diversification ratios for regularly varying and log regularly varying margins with specific copula structures are obtained. Empirical analysis on financial data is provided for highlighting our results. It is a joint work with Wenhao Zhu, Lujun Li, Jiehua Xie and Liulei Sun.

On Consistency of Signatures Using Lasso and Applications in Option Pricing

Ruixun Zhang (Peking University)

Signature transforms are iterated path integrals of continuous and discrete-time time series data, and their universal nonlinearity linearizes the problem of feature selection. This paper revisits some statistical properties of signature transform under stochastic integrals with a Lasso regression framework, both theoretically and numerically. Our study shows that, for processes and time series that are closer to Brownian motion or random walk with weaker inter-dimensional correlations, the Lasso regression is more consistent for their signatures defined by Ito integrals; for mean reverting processes and time series, their signatures defined by Stratonovich integrals have more consistency in the Lasso regression. We apply these results to learning nonlinear functions and option pricing. Our findings highlight the importance of choosing appropriate definitions of signatures and stochastic models in statistical inference and machine learning. This is joint work with Xin Guo, Binnan Wang, and Chaoyi Zhao.