Research course lecturer: Tom Spencer (IAS)

Title: Grassmann integration and Statistical Mechanics

These lectures will describe Grassmann integration and its application to models of statistical mechanics and random matrices. After defining Grassmann integrals and establishing some basic identities, I will present an elementary analysis the density of states for GUE random matrices using a simple version of the supersymmetric formalism. This will be followed by a description of Shcherbina's calculation averages of products determinants of random band matrices. These averages naturally dual to the Heisenberg model.

The two dimensional Ising model with nearest neighbor interaction can be expressed in terms of a "Gibbs weight" which is quadratic in Grassmann fields – that is a free Grassmann field. Hence, the partition function and certain correlations can be explicitly computed. The next to nearest neighbor or plaquette perturbations give rise to quartic perturbation in the Grassmann variables and are not integrable. Nevertheless, we will see that small perturbations of this form are "irrelevlant".

A closely related model is the fermionic hyperbolic sigma model, H(0|2), which has been recently analyzed by Bauerschmidt, Crawford and Helmuth in 3D. It has a continuous symmetry which is broken at low temperature producing Goldstone modes. At low temperature this model it can also be expressed in terms of a small quartic perturbation of a free field in the Grassmann variables Both the perturbed 2D Ising and H(0|2) model are analyzed by renormalization group methods which are briefly sketched.

Research talks:

Dang-Zheng Liu (USTC)

Title: Edge statistics for random band matrices

Abstract: Random band matrices are interpolating models between mean-field Wigner matrices and Anderson models, and are conjectured to exhibit phase transition between Poisson and GOE/GUE statistics as the bandwidth W increases. For Hermitian random band matrices on the d-dimensional lattice $(\mathbb{Z}/{L\mathbb{Z}})^d$, the critical dimension $d_c=6$ and the critical bandwidth $W_c=L^{1-\frac{d}{6}}$ are observed at the spectral edge. As $W\to\infty$ edge statistics are also established in the three spectral regimes when $d<4$. The proof builds upon Sodin's program and new techniques of taming the singularity of Feynman diagrams and graph integrals through a connection to the ϕ^3 model. This is based on join work with Guangyi Zou, arXiv:2401.00492.

Christophe Sabot (Lyon)

Title: Stochastic calculus aspects of the Vertex Reinforced Jump process

Abstract: In this talk we will present some relations between classical computations on exponential functionals of the Brownian motion and some representation of the Vertex Reinforced Jump Process (VRJP). In particular we will show how the Lamperti transformation, the Markov property of the Matsumoto-Yor process have natural generalizations to the multidimensionnal setting involving the representation of the VRJP by a random Schrödinger operator. Besides, a discrete version of the Matsumoto-Yor porperties appears also, in a rather different way, in terms of some natural functionals associated with the one-dimensional VRJP.

Alejandro Ramirez (NYU Shanghai)

Title: Large deviations for random walk in random environment and periodic environments

Abstract: The random walk in random environment is a fundamental mathematical model of statistical mechanics describing a variety of complex large scale phenomena arising from biology, physics and engineering. It was shown in 2017 by Barraquand and Corwin that this model belongs to the KPZ universality class. Furthermore, its large deviation properties show analogies (in a non-directed setting) with corresponding properties of the partition function of polymer models, in particular the distinction between regimes when the quenched and averaged rate functions are equal or not. In this talk we will give a detailed description of the large deviation event that the random walk starting from 0 returns to 0 after a long time, showing the emergence of periodic environments which give rise to the corresponding event. This talk is based on joint works with Alexander Drewitz, Mark Holmes, Alexander Holroyd and Zhicheng Zheng.

Pierre Tarres (NYU Shanghai)

Title: Self-interacting random walk, bayesian statistics and statistical physics

Abstract: We will discuss various models of self-interacting random walks, in particular the Vertex-Reinforced Random Walk (VRRW) and some non-reversible generalizations of the Edge-Reinforced Random Walk (*-ERRW) motivated by Bayesian statistics for variable order Markov Chains. Contrary to the VRRW, the *-ERRW is partially exchangeable in the sense of Diaconis and Freedman (1982), and its mixing measure can be explicitly computed.

Both the VRRW and the *-ERRW can be associated to a continuous process, respectively called the continuous vertex-reinforced random walk (cVRRW) and the *-Vertex Reinforced Random Walk (*-VRJP), both of which are in general not partially exchangeable. The *-VRJP however satisfies several fascinating properties, and in particular a random Schrödinger representation, which will be very useful in the study of recurrence/transience properties.

Based on joint works with S. Bacallado and C. Sabot, and with Shuo Qin.

Fan Yang (Tsinghua)

Title: Delocalization of non-mean-field random matrices

Abstract: Consider two generalizations of the famous Anderson model defined on the d-dimensional integer lattice of linear size L. One is the random band matrix ensemble, whose entries are independent centered complex Gaussian random variables such that H_{xy} is nonzero only when $|x-y|$ is less than the band width W. Another one is the block Anderson model, which replaces the i.i.d. diagonal potential in Anderson model by an i.i.d. diagonal block potential with coupling strength parameter $\lambda>0$ and blocks of linear size W. Both of these models are non-mean-field models with W describing the length of local interactions. Moreover, they are conjectured to exhibit Anderson transitions as W or λ varies. In this talk, I will discuss some of our recent works on the delocalization of these two models when $d\ge 7$ and $W\geq L^\delta$ for a small constant $\delta>0$. Based on joint works with Changji Xu, Horng-Tzer Yau and Jun Yin.

Qi Zhou (Nankai)

Title: Phase transition and mobility edges

Abstract: The disorder systems host three types of fundamental quantum states, known as the extended, localized, and critical states, of which the critical states remain being much less explored. In this talk, we propose several classes of exactly solvable models which host exact mobility edges (MEs) separating extended states and localized states, MEs separating localized states from critical states, MEs separating critical states and extended states. We will also discuss their rigorous proof.