The next-to-top term in knot Floer homology

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- An overview of knot Floer homology
- Is the second term nontrivial?
- A bound on the number of fixed points
- Characterizing right-veering homeomorphisms

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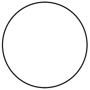

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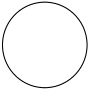

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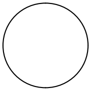

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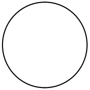

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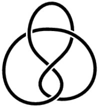

figure–8 knot

Seifert surfaces

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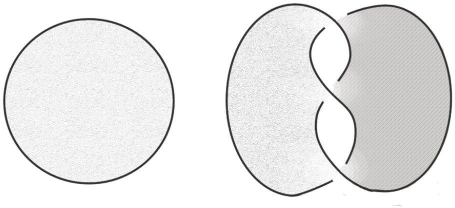

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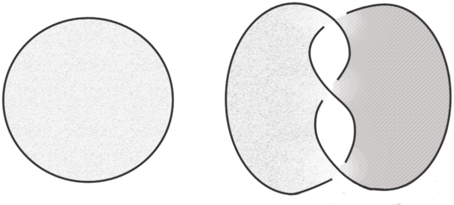

The (Seifert) genus of K is defined to be

$$g(K) = \min\{g(F) | \ F \text{ is a Seifert surface for } K\}.$$

Fibered knots

If the complement of K is a surface bundle over the circle, then K is called a fibered knot. The unknot, the trefoils and the figure–8 knot are fibered knots.

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In other words, there exists a surface-with-boundary F and an orientation-preserving homeomorphism $h : F \to F$, such that

$$S^3 \setminus \mathrm{int}(\mathrm{Nd}(K)) \cong F \times [0,1]/\sim,$$

where

$$(x, 1) \sim (h(x), 0), \text{ for any } x \in F.$$

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We necessarily have $g(F) = g(K)$.

Alexander polynomial

The powerful knot invariant Alexander polynomial (Alexander 1928) can be defined and computed as follows.

$$
\begin{aligned}
\Delta_{\mathrm{unknot}}(t) &= 1, \\
\Delta_{L_+} - \Delta_{L_-} &= (t^{\frac{1}{2}} - t^{-\frac{1}{2}})\Delta_{L_0}.
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The second formula is the “skein relation" (Alexander 1928, Conway 1969), where $L_\pm, L_0$ denote the three oriented links that differ at exactly one crossing.

Examples

When K is a knot,

$$\Delta_K(t) \in \mathbb{Z}[t, t^{-1}].$$

In general, for a link L,

$$\Delta_L(t) \in \mathbb{Z}[t^{\frac{1}{2}}, t^{-\frac{1}{2}}].$$

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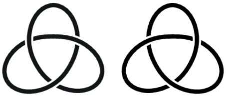

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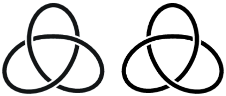

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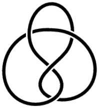

$$\Delta_{\text{figure–8}}(t) = -t^{-1} + 3 - t.$$

The Alexander polynomial of a fibered knot

Suppose that K is a fibered knot with monodromy $h : F \to F$. Then $\Delta_K(t)$ is, up to a factor $\pm t^{-g}$, the characteristic polynomial of

$$h_* : H_1(F) \to H_1(F).$$

Namely,

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Let

$$\Delta_K(t) = \sum_{i=-g}^{g} a_i t^i,$$

then

$$a_g = \pm 1, \quad a_{g-1} = -a_g \mathrm{tr}(h_*).$$

Genus bound from Δ_K

There is a similar interpretation of Δ_K even when K is non-fibered. From this interpretation, one can deduce that the degree of Δ_K is bounded by the genus:

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The above inequality is not sharp. In fact, there are infinitely many knots with $\Delta_K = 1$.

Knot Floer homology

Knot Floer homology is a categorification of the Alexander polynomial. When K is a knot in S^3, the knot Floer homology is a finitely generated bigraded abelian group

$$\widehat{HFK}(S^3, K) = \bigoplus_{a,m\in\mathbb{Z}} \widehat{HFK}_m(S^3, K, a).$$

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The Euler characteristic of $\widehat{HFK}(S^3, K)$ gives rise to the Alexander polynomial, namely

$$\sum_a \chi(\widehat{HFK}_*(S^3, K, a)) \cdot t^a = \Delta_K(t).$$

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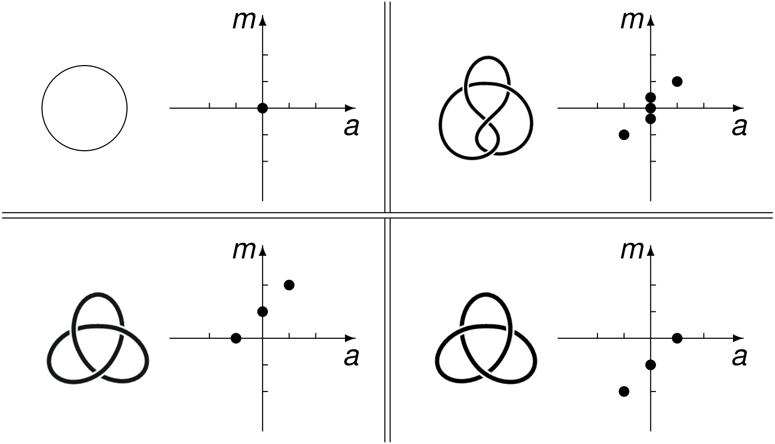
This fact can be generalized to a symmetry in $\widehat{HFK}$:

$$\widehat{HFK}_*(S^3, K, a; \mathbb{Q}) \cong \widehat{HFK}_{*-2a}(S^3, K, -a; \mathbb{Q}).$$



The knot Floer homologies of the first 4 knots

Here a black dot stands for a copy of $\mathbb{Z}$.

The topmost term in knot Floer homology

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Theorem (Ozsváth–Szabó)

Suppose K is a knot in S^3, $g(K)$ is its genus. Then

$$g(K) = \max\{a \mid \widehat{HFK}(S^3, K, a) \neq 0\}.$$

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Theorem (Ozsváth–Szabó, Ghiggini+Ni)

Suppose K is a knot in S^3 with genus g. Then

$$\widehat{HFK}(S^3, K, g) \cong \mathbb{Z}$$

if and only if K is fibered.

Knots in general 3-manifolds

Let Y be a closed, oriented, connected 3-manifold, $K \subset Y$ be a null-homologous knot. One can define knot Floer homology for the pair (Y, K):

$$\widehat{HFK}(Y, K) = \bigoplus_{a \in \mathbb{Z}} \widehat{HFK}(Y, K, a).$$

The previous two theorems still hold true in the general case.

- An overview of knot Floer homology
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What about other terms?

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In this talk, we will focus on the next-to-top term, or the second term,

$$\widehat{HFK}(S^3, K, g(K) - 1).$$

Is the second term nontrivial?

Conjecture (Baldwin–Vela-Vick)

For any nontrivial knot $K \subset S^3$, one has

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A stronger conjecture is:

Conjecture (Sivek)

For any nontrivial knot $K \subset S^3$, one has

$$\mathrm{rank}\,\widehat{HFK}(S^3, K, g(K) - 1) \geq \mathrm{rank}\,\widehat{HFK}(S^3, K, g(K)).$$

Known results

Sivek's conjecture has been proved for

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- fibered knots, including all L-space knots (Baldwin–Vela-Vick)

Theorem (Ni)

Let $K \subset S^3$ be a knot with genus $g > 0$. Suppose that $\widehat{HFK}(S^3, K, g)$ is supported in a single $\mathbb{Z}/2\mathbb{Z}$–grading. Then for any $m \in \mathbb{Z}$, we have

$$\mathrm{rank}\widehat{HFK}_{m-1}(S^3, K, g-1) \geq \mathrm{rank}\widehat{HFK}_m(S^3, K, g).$$

This result contains all known cases of the conjecture.

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The Lefschetz–Hopf index formula

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We can isotope h so that all fixed points of h are isolated, then the Lefschetz–Hopf index formula says

$$\begin{aligned}\sum_{x \in \mathrm{Fix}(h)} \mathrm{index}(h, x) &= 1 - \mathrm{tr}(h_*) \\ &= 1 + a_g a_{g-1} \\ &= 1 + a_g \chi(\widehat{HFK}(S^3, K, g-1)),\end{aligned}$$

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In particular, if $a_g a_{g-1} \neq -1$, then any map isotopic to h has at least a fixed point.

The bound on the number of fixed points

Theorem (Ni)

Let $K \subset Y$ be a fibered knot with fiber F and monodromy h. If

$$\mathrm{rank}\widehat{HFK}(Y, K, g-1) = r,$$

then h is freely isotopic to a diffeomorphism with at most $r-1$ fixed points.

In particular, when $r = 1$, h is freely isotopic to a diffeomorphism with no fixed points. L-space knots are examples of such knots. Baldwin–Hu–Sivek proved this result for knots with the same knot Floer homology as the torus knot $T(5, 2)$.

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A similar result was also proved by Ghiggini–Spano using a completely different framework.

Why is knot Floer homology related to fixed points?

Let Σ be a closed oriented surface with an area form ω, $\varphi : \Sigma \to \Sigma$ be an area-preserving map. Then φ is a symplectomorphism of the symplectic 2–manifold (Σ, ω).

Why is knot Floer homology related to fixed points?

Let Σ be a closed oriented surface with an area form ω, $\varphi : \Sigma \to \Sigma$ be an area-preserving map. Then φ is a symplectomorphism of the symplectic 2–manifold (Σ, ω).

One can define a symplectic Floer homology $HF^{\mathrm{symp}}(\varphi)$. The chain complex is freely generated by the fixed points of φ if all fixed points are non-degenerate. Moreover, the isomorphism class of $HF^{\mathrm{symp}}(\varphi)$ only depends on the mapping class of φ (Seidel).

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It has been conjectured that the second term of $\widehat{HFK}(Y, K)$ should be related to a suitable version of the above symplectic Floer homology. Our proof affirms this conjecture.

Different versions of Floer homologies

Using an argument of Baldwin–Hu–Sivek and a trick in Heegaard Floer homology, we can relate the following Floer homologies:

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\begin{aligned}
& \widehat{HFK}(Y, K, g(K) - 1) \\
\approx\ & \text{Heegaard Floer homology of a closed fibered 3-manifold } Z \\
\cong\ & \text{Monopole Floer homology of } Z \text{ (Kutluhan–Lee–Taubes)} \\
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A complete computation of $HF^{\mathrm{symp}}(\varphi)$ was given by Cotton-Clay. We then use Jiang–Guo's work on the Nielsen fixed point theory of surfaces to get our bound.

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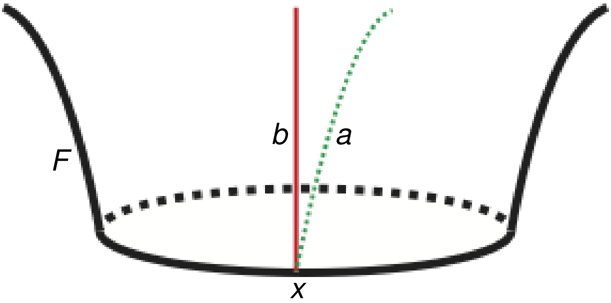
To the right



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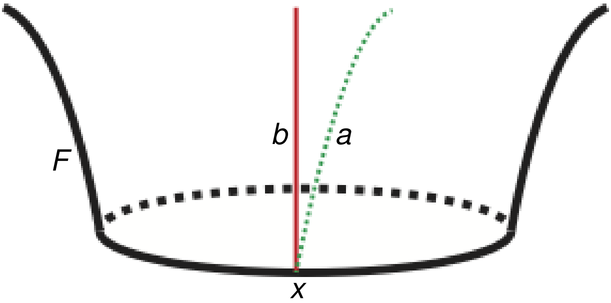

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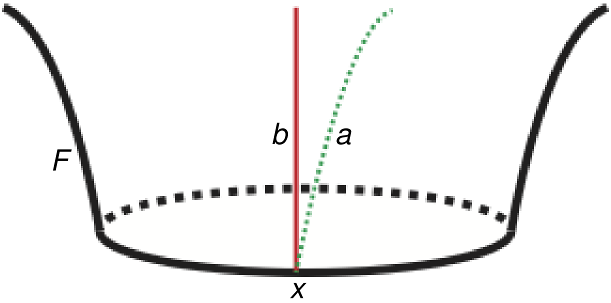

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- either a is isotopic to b rel ∂,
- or after an isotopy rel ∂ to minimize $|a \cap b|$, $(a \cap U) \cap \{x\}$ lies in the right component of $U \setminus b$ for a small neighborhood U of x.

Honda–Kazez–Matić introduced the following concept, which enables them to characterize tight contact structures in terms of mapping classes.

Definition

Let F be a compact oriented surface with boundary, $h\colon F \to F$ be a diffeomorphism that restricts to the identity map on ∂F. Then h is right-veering if for every $x \in \partial F$ and every properly embedded arc $a \subset F$ with $x \in a$, the image $h(a)$ is to the right of a at x. Similarly, we can define left-veering diffeomorphisms.

Right-veering diffeomorphisms

Honda–Kazez–Matić introduced the following concept, which enables them to characterize tight contact structures in terms of mapping classes.

Definition

Let F be a compact oriented surface with boundary, $h\colon F \to F$ be a diffeomorphism that restricts to the identity map on ∂F. Then h is right-veering if for every $x \in \partial F$ and every properly embedded arc $a \subset F$ with $x \in a$, the image $h(a)$ is to the right of a at x. Similarly, we can define left-veering diffeomorphisms.

The identity map is the only map which is both right-veering and left-veering.

More structures on $\widehat{HFK}$

Let $K \subset Y$ be a null-homologous knot. There is a homomorphism

$$(\partial_z)_* : \widehat{HFK}(Y, K, a) \to \widehat{HFK}(Y, K, a-1), \quad a \in \mathbb{Z}.$$

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We will consider two summands of $(\partial_z)_*$:

$$\partial^{\mathrm{top}} : \widehat{HFK}(Y, K, g) \to \widehat{HFK}(Y, K, g-1),$$

$$\partial^{\mathrm{bot}} : \widehat{HFK}(Y, K, 1-g) \to \widehat{HFK}(Y, K, -g).$$

A characterization of right-veering diffeomorphisms

Theorem (Baldwin–Vela-Vick)

Let $K \subset Y$ be a fibered knot with fiber F, and let $h : F \to F$ be the monodromy. If $\partial^{\mathrm{top}} = 0$, then h is right-veering.

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The converse is also true.

Theorem (Baldwin–Ni–Sivek)

Let $K \subset Y$ be a fibered knot with fiber F, and let $h : F \to F$ be the monodromy. If h is right-veering, then $\partial^{\text{top}} = 0$.

A characterization of right-veering diffeomorphisms, continued

Any map $h : F \to F$ with $h|_{\partial F} = \mathrm{id}_{\partial F}$ falls into one of four classes:

$$\mathrm{id}, \mathrm{RV} \text{ not } \mathrm{id}, \mathrm{LV} \text{ not } \mathrm{id}, \text{neither RV nor LV.}$$

A characterization of right-veering diffeomorphisms, continued

Any map $h : F \to F$ with $h|_{\partial F} = \mathrm{id}_{\partial F}$ falls into one of four classes:

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Using the previous theorem and the symmetry in $\widehat{HFK}$, the above four classes can be distinguished by the information from $\widehat{HFK}$:

h	∂^{top}	∂^{bot}
id	$= 0$	$= 0$
RV, $\neq \mathrm{id}$	$= 0$	$\neq 0$
LV, $\neq \mathrm{id}$	$\neq 0$	$= 0$
neither RV nor LV	$\neq 0$	$\neq 0$

Thank you!