



北京大学
PEKING UNIVERSITY

PEKING UNIVERSITY MATHEMATICS FORUM

北大数学校友论坛

北京大学镜春园82号院甲乙丙楼
二层报告厅

应用数学专场 7.31-8.1
基础数学专场 8.2-8.5

SCHEDULE

会议日程

应用数学专场

7.31 周一

| | | | |
|-------------|-------|---|--|
| 8:45-9:00 | 开幕式 | | 主持：张磊 |
| 9:00-9:40 | 1 | 鲁剑锋 (Duke University) | Analysis of score-based generative models |
| 9:40-10:20 | 2 | 蔡振宁 (National University of Singapore) | 复值朗之万方法的性质分析与稳定化方案 |
| | | 主持：李铁军 | |
| 10:20-10:40 | 茶歇，合影 | | |
| 10:40-11:20 | 3 | 谢季纯 (Duke University) | SifiNet: A robust and accurate method to identify feature gene sets and annotate cells |
| 11:20-12:00 | 4 | 钟声 (University of California, San Diego) | Toward a Blood-Based Biomarker for Early Detection of Alzheimer's Disease |
| | | 主持：葛颢 | |
| 12:00-14:00 | 午餐 | | |
| 14:00-14:40 | 5 | 杨志坚 (Wuhan University) | Improved Analysis of PINNs: Alleviate the CoD for Compositional Solutions |
| 14:40-15:20 | 6 | 胡婧玮 (University of Washington) | Collisional Particle-In-Cell Method for the Vlasov-Maxwell-Landau system |
| 15:20-16:00 | 7 | 刘歆 (Chinese Academy of Sciences) | Constraint Dissolving Approaches for a Class of Riemannian Optimization Problems |
| | | 主持：李若 | |
| 16:00-16:20 | 茶歇 | | |
| 16:20-17:00 | 8 | 胡凯博 (University of Oxford) | The Bernstein-Gelfand-Gelfand machinery and applications |
| 17:00-17:40 | 9 | 王涵 (Institute of Applied Physics and Computational Mathematics) | Deep learning in molecular dynamics simulations |
| | | 主持：周珍楠 | |

8.1 周二

| | | | | |
|-------------|----|---|--|---------|
| 9:00-9:40 | 10 | 林霖 (University of California, Berkeley) | Quantum algorithms for eigenvalue problems | 主持: 胡俊 |
| 9:40-10:20 | 11 | 孙若愚 (The Chinese University of Hong Kong, Shenzhen) | Why Are Modern Neural Networks So Large? An Optimization Landscape Perspective | |
| 10:20-10:40 | | | 茶歇 | |
| 10:40-11:20 | 12 | 宋洁 (Peking University) | Design and Analysis of Decentralized Zeroth-order Algorithms for Complex Systems | 主持: 葛颢 |
| 11:20-12:00 | 13 | 马宗明 (Yale University) | Dataset matching and its applications in single-cell data integration | |
| 12:00-14:00 | | | 午餐 | |
| 14:00-14:40 | 14 | 唐晓弦 (Beihang University) | Multistationarity of Small Zero-One Networks | 主持: 文再文 |
| 14:40-15:20 | 15 | 周栋焯 (Shanghai Jiao Tong University) | 计算神经科学: 连接脑科学与人工智能的桥梁 | |
| 15:20-15:30 | | | 茶歇 | |
| 15:30-17:00 | | | 交流会 | 主持: 张磊 |

基础数学专场

8.2 周三

| | | |
|-------------|---|---|
| 9:00-9:20 | | 开幕式 |
| 9:20-10:20 | 倪忆 (California Institute of Technology) | The next-to-top term in knot Floer homology |
| 10:20-11:00 | | 合影, 茶歇 |
| 11:00-12:00 | 朱歆文 (Stanford University) | The p-adic Borel Hyperbolicity of A_g |
| 12:00-14:00 | | 午餐 |
| 14:00-15:00 | 刘志鹏 (University of Kansas) | Some exact formulas of the KPZ fixed point and directed landscape |
| 15:00-15:30 | | 茶歇 |
| 15:30-16:30 | 姚珏 (National University of Singapore) | Small scale formations in fluid equations with gravity |

8.3 周四

| | | |
|-------------|---|---|
| 9:30-10:30 | 许晨阳 (Princeton University) | Kähler-Einstein metric, K-stability and moduli spaces |
| 10:30-11:00 | | 茶歇 |
| 11:00-12:00 | 余越 (California Institute of Technology) | Moduli space of non-archimedean holomorphic disks |
| 12:00-14:00 | | 午餐 |
| 14:00-15:00 | 刘钢 (East China Normal University) | Gromov-Hausdorff convergence of Kähler manifolds |
| 15:00-15:30 | | 茶歇 |
| 15:30-16:30 | 徐宙利 (University of California, San Diego) | The Adams differentials on the classes h_j^3 |
| 16:45-17:45 | 周鑫 (Cornell University) | Some recent development in minimal surface theory |

8.4 周五

| | | |
|-------------|--|--|
| 9:30-10:30 | 张伟 (Massachusetts Institute of Technology) | Algebraic cycles over number fields and L-functions: conjectures and results |
| 10:30-11:00 | 茶歇 | |
| 11:00-12:00 | 高紫阳 (University of Hannover) | Sparsity of Rational and Algebraic Points |
| 12:00-14:00 | 午餐 | |
| 14:00-15:00 | 张瑞祥 (University of California, Berkeley) | Fourier restriction type problems: New developments in the last 15 years |
| 15:00-15:30 | 茶歇 | |
| 15:30-16:30 | 王虹 (University of California, Los Angeles) | Incidence estimates and applications |

8.5 周六

| | | |
|-------------|---|--|
| 9:30-10:30 | 何旭华 (University of Hong Kong) | Towards a Geometric Theory of Characters |
| 10:30-11:00 | 茶歇 | |
| 11:00-12:00 | 恽之玮 (Massachusetts Institute of Technology) | Functions on the commuting scheme via Langlands duality |
| 12:00-14:00 | 午餐 | |
| 14:00-15:00 | 王博潼 (University of Wisconsin-Madison) | Perverse sheaves and positivity of Euler characteristics |
| 15:00-15:30 | 茶歇 | |
| 15:30-16:30 | 申皓 (University of Wisconsin-Madison) | Stochastic quantization of Yang-Mills |

TITLES AND ABSTRACTS

报告的题目和摘要

应用数学专场

鲁剑锋 (Duke University)

Title: Analysis of score-based generative models

Abstract: Score-based generative modeling (SGM) is a highly successful approach for learning a probability distribution from data and generating further samples, based on learning the score function (gradient of log-pdf) and then using it to simulate a stochastic differential equation that transforms white noise into the data distribution.

In this talk, we will discuss some recent works in convergence analysis of SGM and related methods. In particular, we established convergence of SGM applying to any distribution with bounded 2nd moment, relying only on a L^2 -accurate score estimates, with polynomial dependence on all parameters and no reliance on smoothness or functional inequalities

蔡振宁 (National University of Singapore)

Title: 复值朗之万方法的性质分析与稳定化方案

Abstract: 复值朗之万方法是用于缓解格点量子场论中路径积分计算中符号问题的一种数值方法。从数学角度来看，它可以被视为计算高维振荡积分的一种数值工具，适用于具有系综平均形式的情况。该方法形式上是传统采样点朗之万方法的自然推广。尽管复值朗之万方法在上世纪八十年代就被提出，但由于常常收敛到不正确的结果，它在提出后很长时间内未能被完全理解且鲜有实际应用。约十年前，学者们基本厘清了复值朗之万方法未能正确收敛的原因，并在结合规范不变性后显著提升了该方法的数值表现。这使得它在量子色动力学、超弦理论等领域内得到较为成功的应用。在本次报告中，我将从数学角度探讨该方法的性质和导致失败的原因，并讨论如何运用稳定化技巧进一步扩大其应用范围。

谢季纯 (Duke University)

Title: SifiNet: A robust and accurate method to identify feature gene sets and annotate cells

Abstract: SifiNet is a robust and accurate computational pipeline for identifying distinct gene sets, extracting and annotating cellular subpopulations, and elucidating intrinsic relationships among these subpopulations. Uniquely, SifiNet bypasses the cell clustering stage, commonly integrated into other cellular annotation pipelines, thereby circumventing potential inaccuracies in clustering that may compromise subsequent analyses. Consequently, SifiNet has demonstrated superior performance in multiple experimental datasets compared with other state-of-the-art methods. SifiNet can analyze both single-cell RNA and ATAC sequencing data, thereby rendering comprehensive multiomic cellular profiles. It is conveniently available as an open-source R package.

钟声 (University of California, San Diego)

Title: Toward a Blood-Based Biomarker for Early Detection of Alzheimer's Disease

Abstract: The extracellular RNAs (exRNAs) from human biofluid have recently been systematically characterized. However, the correlations of biofluid exRNA levels and human diseases remain largely untested. Here, considering the unmet need for presymptomatic biomarkers of sporadic Alzheimer's disease (AD), we leveraged the recently developed SILVER-seq (small-input liquid volume extracellular RNA sequencing) technology to analyze exRNA from a longitudinal collection of human plasma samples. When we required statistical significance with multiple testing adjustments, phosphoglycerate dehydrogenase (PHGDH) was the only gene that exhibited consistent upregulation in AD brain transcriptomes from 3 independent cohorts and an increase in AD plasma as compared to controls. We validated PHGDH's serum exRNA and brain protein expression increases in AD by using 5 additional research cohorts. Furthermore, human hippocampal PHGDH protein expression level is reversely correlated with the person's cognitive ability. These data suggest the potential utilities of plasma exRNA levels for screening sporadic AD.

杨志坚 (Wuhan University)

Title: Improved Analysis of PINNs: Alleviate the CoD for Compositional Solutions

Abstract: 我们利用偏微分方程解的特殊结构, 证明了 PINNs 方法的收敛阶, 该结果可以减轻维数灾难。具体而言, 当方程解由若干低自由度函数复合而成时, 我们分别证明了深度神经网络的逼近误差和统计误差均仅依赖于方程解的内在自由度。从而得到 PINNs 方法求解高维二阶椭圆方程的收敛阶仅依赖于方程解的内在自由度。此外, 我们还提供了两个数值例子, 以验证理论上的发现。

胡婧玮 (University of Washington)

Title: Collisional Particle-In-Cell Method for the Vlasov-Maxwell-Landau system

Abstract: In this talk we will present an extension of the classical Particle-In-Cell (PIC) method for plasmas which can account for the collisional effects modelled by the Landau operator. The method is derived from the gradient-flow formulation of the Landau equation, thereby preserving the collision invariants and entropy structure. We will discuss the derivation and implementation of the method, as well as several numerical examples to showcase the effects of collisions in plasma dynamics.

刘歆 (Chinese Academy of Sciences)

Title: Constraint Dissolving Approaches for a Class of Riemannian Optimization Problems

Abstract: We propose constraint dissolving approaches for optimization problems over a class of Riemannian manifolds. In these proposed approaches, solving a Riemannian optimization problem is transferred into the unconstrained minimization of a constraint dissolving function named CDF. Different from existing exact penalty functions, the exact gradient and Hessian of CDF are easy to compute. We study the theoretical properties of CDF and prove that the original problem and CDF have the same first-order and second-order stationary points, local minimizers, and ojasiewicz exponents in a neighborhood of the feasible region. Remarkably, the convergence properties of our proposed constraint dissolving approaches can be directly inherited from the existing rich results in unconstrained optimization. Therefore, the proposed constraint dissolving approaches build up short cuts from unconstrained optimization to Riemannian optimization. Several illustrative examples further demonstrate the potential of the proposed approaches.

胡凯博 (University of Oxford)

Title: The Bernstein-Gelfand-Gelfand machinery and applications

Abstract: In this talk, we first review the de Rham complex and the finite element exterior calculus, a cohomological framework for structure-preserving discretisation of PDEs. From de Rham complexes, we derive other complexes with applications in elasticity, geometry and general relativity. The derivation, inspired by the Bernstein-Gelfand-Gelfand (BGG) construction, also provides a general machinery to establish results for tensor-valued problems (e.g., elasticity) from de Rham complexes (e.g., electromagnetism and fluid mechanics). We discuss some applications and progress in this direction.

王涵 (Institute of Applied Physics and Computational Mathematics)

Title: Deep learning in molecular dynamics simulations

Abstract: In this report, we first introduce the basic concepts, methods, and enormous application value of molecular dynamics simulations. We point out the two challenges faced by molecular dynamics simulations: potential energy modeling and rare event sampling. In terms of potential energy modeling, we introduce the deep potential, a method that combines physical laws, deep learning, and high-performance computing to solve the dilemma of accuracy and efficiency. Specifically, we introduce how to incorporate attention mechanisms into the pre-trained deep potential models, aiming to solve the rapid training and generalization problems on extremely large datasets. In terms of rare event sampling, we introduce the adaptive reinforced dynamics method, which overcomes the curse of dimensionality problem in the enhanced sampling algorithms. We will demonstrate through examples that this method can effectively discover metastable configurations in a collective variable space with more than 100 dimensions.

林霖 (University of California, Berkeley)

Title: Quantum algorithms for eigenvalue problems

Abstract: The problem of finding the smallest eigenvalue of a Hermitian matrix (also called the ground state energy) has wide applications in quantum physics. In this talk, I will first briefly introduce the mathematical setup of quantum algorithms, and discuss how to use textbook quantum algorithms to tackle this problem. I will then introduce a new quantum algorithm that can significantly and provably reduce the circuit depth for solving this problem (the reduction can be around two orders of magnitude). This algorithm reduces the requirement on the maximal coherent time for the quantum computer, and can therefore be suitable for early fault-tolerant quantum devices. No prior knowledge on quantum algorithms is necessary for understanding most parts of the talk.

孙若愚 (The Chinese University of Hong Kong, Shenzhen)

Title: Why Are Modern Neural Networks So Large? An Optimization Landscape Perspective

Abstract: Modern neural networks are getting larger and larger: compared to LeNet in 1980's which has 60k parameters, recent networks have millions of parameters (e.g. ResNet) or even trillions of parameters (e.g. GPT3). Training such large models consumes lots of power and time. Why do researchers use such large models? While one obvious reason is to increase the representation power, we argue that a critical reason is to smooth the optimization landscape so that local search algorithms can converge to better solutions. Formally, we prove that for unregularized loss function, the loss landscape of narrow neural networks may contain suboptimal local basins, while the loss landscape of wide enough networks does not. This establishes a phase transition between narrow and wide networks. Further, for regularized loss function, we prove that for 2-layer networks with ReLU activation, the loss landscape contains no sub-optimal local minima. We provide empirical evidences that support our finding.

宋洁 (Peking University)

Title: Design and Analysis of Decentralized Zeroth-order Algorithms for Complex Systems

Abstract: It is very challenging to describe and model the dynamics of practical complex systems. We can hardly obtain enough information about the dynamics except for the zeroth-order information of the systems, let alone solve the optimization problem effectively and efficiently in a decentralized manner. Thus, we design zeroth-order feedback-based gradient descent algorithms (ZFGD) to solve the problems in power systems without the information of dynamics. First, we introduce gradient estimations to design a decentralized algorithm and analyze its performance. Then, another random ZFGD algorithm (RZFGD) based on coordinate descent is proposed to considerably reduce the number of required gradient estimations. We prove RZFGD can achieve the same convergence rate as the unconstrained problem or without gradient estimations in the distributed demand response problem. The empirical experiments are also conducted to validate the performance of the proposed algorithms.

马宗明 (Yale University)

Title: Dataset matching and its applications in single-cell data integration

Abstract: We study one-way matching of a pair of datasets with low-rank signals. Under a stylized model, we first derive information-theoretic limits of matching under a mismatch proportion loss. We then show that linear assignment with projected data achieves fast rates of convergence and sometimes even rate optimality for this task. Built upon this theoretical understanding, we propose two matching algorithms for the integration of data collected from different single-cell modalities. We showcase their applications on a COVID-19 and a HuBMAP intestine data integration example, respectively.

This talk is based on joint works with Shuxiao Chen, Bokai Zhu, Sizun Jiang, Garry P. Nolan, and Nancy R. Zhang.

唐晓弦 (Beihang University)

Title: Multistationarity of Small Zero-One Networks

Abstract: Zero-one networks are very common in cell signaling. In this talk, we will present the recent progress on the multistationarity problem of small zero-one networks. First, we show that if a zero-one network admits multistationarity, then the rank is at least three. Also, we show that the smallest multistationary zero-one network is rank-three, and it has three species and five reactions. Second, we classify all rank-one and rank-two zero-one networks according to if they admit one (stable) steady state or not.

周栋焯 (Shanghai Jiao Tong University)

Title: 计算神经科学：连接脑科学与人工智能的桥梁

Abstract: 本报告先介绍脑科学作为实验推动的科学领域，对于数学定量刻画（建模、分析和模拟）的迫切需求以及计算神经科学作为交叉学科研究领域与脑科学、人工智能的关系，然后介绍我们课题组在计算神经科学领域开展的相关工作，强调数学与神经科学等其它学科交叉合作的重要性以及开展交叉科学研究的模式和挑战。

基础数学专场

倪忆

Title: The next-to-top term in knot Floer homology

Abstract: Knot Floer homology is a knot invariant which categorifies the Alexander polynomial. This invariant contains a lot of information about the topology of the knot. For example, the topmost Alexander grading of the knot Floer homology of a knot is the Seifert genus of this knot, and the summand of the knot Floer homology at this grading has rank one if and only if the knot is fibered. In recent years, it became clear that the summand of the knot Floer homology at the next-to-top Alexander grading also contains information about the topology of the knot. In this talk, we will address the question whether this summand is nontrivial, and discuss the information contained in this summand about the monodromy when the knot is fibered.

朱歆文

Title: The p-adic Borel Hyperbolicity of A_g

Abstract: A theorem of Borel says that any holomorphic map from a smooth complex algebraic variety to a smooth arithmetic variety is automatically an algebraic map. The key ingredient is to show that any holomorphic map from the punctured disc to the arithmetic variety has no essential singularity. I will discuss some work towards a p-adic analogue of this theorem for moduli of abelian varieties. Joint with Abhishek Oswal and Ananth Shankar.



刘志鹏

Title: Some exact formulas of the KPZ fixed point and directed landscape

Abstract: In the past twenty years, there have been huge developments in the study of the Kardar-Parisi-Zhang (KPZ) universality class, which is a broad class of physical and probabilistic models including one-dimensional interface growth processes, interacting particle systems and polymers in random environments, etc. It is broadly believed and partially proved, that all the models share the universal scaling exponents and have the same asymptotic behaviors. The height functions of models in the KPZ universality class are expected to converge to a limiting space-time fluctuation field, the KPZ fixed point. Moreover, there is a random “directed metric” on the space-time plane that is expected to govern all the models in the KPZ universality class. This “directed metric” is called the directed landscape. Both the KPZ fixed point and the directed landscape are central objects in the study of the KPZ universality class, while they were only characterized/constructed very recently [MQR21, DOV18].

In this talk, we will discuss some exact formulas of distributions in these two random fields and their analogs in the periodic domain. These exact formulas are given by Fredholm determinant or their analogs. We will show some surprising probabilistic properties of the KPZ fixed point and the directed landscape using the exact formulas. Some of the results are based on joint work with Jinho Baik, Yizao Wang, and Ray Zhang.

姚珧

Title: Small scale formations in fluid equations with gravity

Abstract: Abstract: In this talk, we discuss some PDEs that describe fluid motion under the influence of gravity, including the incompressible porous media equation and incompressible Boussinesq equation in two dimensions. Using an interplay between various monotone and conserved quantities, we construct rigorous examples of small scale formations as time goes to infinity. These growth results work for a broad class of initial data, where we only require certain symmetry and sign conditions. As an application, we also construct solutions to the 3D axisymmetric Euler equation whose velocity has infinite-in-time growth. (Based on joint works with Alexander Kiselev and Jaemin Park).

许晨阳

Title: Kähler-Einstein metric, K-stability and moduli spaces

Abstract: The question of whether a smooth complex variety with a positive first Chern class, called a Fano variety, has a Kähler-Einstein metric has been a major topic in complex geometry since the 1980s. In the last decade, algebraic geometry, or more specifically higher dimensional geometry has played a surprising role in advancing our understanding of this problem. The interplay between complex geometry and algebraic geometry has also provided deep insights into higher dimensional algebraic geometry itself, peaked by the construction of a projective moduli space that parametrizes Fano varieties. More precisely, the moduli space parametrizes Fano variety satisfying the stability condition which is used to characterize the existence of a Kähler-Einstein metric - known as K-stability. In the lecture, I will explain the main ideas behind the recent progress of the field.

余越

Title: Moduli space of non-archimedean holomorphic disks

Abstract: I will describe the moduli space of non-archimedean holomorphic disks in affine log Calabi-Yau varieties, which is foundational to the non-archimedean mirror symmetry program. I will discuss boundary conditions, smoothness, dimension and properness. Smoothness relies on the non-archimedean deformation theory joint with M. Porta. Properness relies on formal models and Temkin's theory of reduction of germs. Work in progress with S. Keel.

刘钢

Title: Gromov-Hausdorff convergence of Kähler manifolds

Abstract: We survey recent progress on Gromov-Hausdorff convergence of Kähler manifolds with geometric applications.

徐宙利

Title: The Adams differentials on the classes h_j^3

Abstract: In filtration 1 of the Adams spectral sequence, using secondary cohomology operations, Adams computed the differentials on the classes h_j , resolving the Hopf invariant one problem. In Adams filtration 2, using equivariant and chromatic homotopy theory, Hill--Hopkins--Ravenel proved that the classes h_j^2 support non-trivial differentials for $j \geq 7$, resolving the celebrated Kervaire invariant one problem.

I will talk about joint work with Robert Burklund: In Adams filtration 3, we prove an infinite family of non-trivial d_4 -differentials on the classes h_j^3 for $j \geq 6$, confirming a conjecture of Mahowald. Our proof uses two different deformations of stable homotopy theory – C-motivic stable homotopy theory and F_2 -synthetic homotopy theory – both in an essential way.

周鑫

Title: Some recent development in minimal surface theory

Abstract: We will present some recent progress on two problems in minimal surface theory posed by S. T. Yau in 1982. In particular, we will discuss the existence of infinitely many closed minimal hypersurfaces in a closed Riemannian manifold and the existence of four closed minimal two-spheres in a Riemannian three-sphere.

张伟

Title: Algebraic cycles over number fields and L-functions: conjectures and results.

Abstract: Algebraic cycles are among the most fundamental mathematical objects. Those defined over number fields are of particular interest. There are also analytic invariants, the Hasse-Weil L-functions, attached to algebraic varieties over number fields. We will review the history of a few conjectures on the connection between them, focusing on that of Tate and Birch—Swinnerton-Dyer. We will then state a few recent results towards these conjectures, proved by studying automorphic period integral and arithmetic intersection theory on Shimura varieties.

高紫阳

Title: Sparsity of Rational and Algebraic Points

Abstract: It is a fundamental question in mathematics to find rational solutions to a given system of polynomials, and in modern language this question translates into finding rational points in algebraic varieties. This question is already very deep for algebraic curves defined over \mathbb{Q} . An intrinsic natural number associated with the curve, called its genus, plays an important role in studying the rational points on the curve. In 1983, Faltings proved the famous Mordell Conjecture (proposed in 1922), which asserts that any curve of genus at least 2 has only finitely many rational points. Thus the problem for curves of genus at least 2 can be divided into several grades: finiteness, bound, uniform bound, effectiveness. An answer to each grade requires a better understanding of the distribution of the rational points.

In my talk, I will explain the historical and recent developments of this problem according to the different grades. Another important topic on studying points on curves is the torsion packets. This topic goes beyond rational points. I will also discuss briefly about it in my talk.

张瑞祥

Title: Fourier restriction type problems: New developments in the last 15 years

Abstract: Fourier 限制型问题是调和分析中的一大类重要问题。给定欧氏空间中一个函数，其 Fourier 变换支在某个弯曲的子流形上，(Fourier) 限制型问题常关心此种函数的某些 L^p 范数的最佳估计。除调和分析外，限制型问题与解析数论、偏微分方程、几何测度论、数学物理等都有密切的联系。最近十五年来，数学工作者们在限制型问题上取得了许多进展。在报告中我将介绍这类问题，选讲十五年来的一些新突破，最后再展望一些该领域未来值得关注的方向。

王虹

Title: Incidence estimates and applications

Abstract: Let P be a set of points and L be a set of lines in the plane, what can we say about the number of incidences between P and L , $I(P, L) := |\{(p, l) \in P \times L, p \in l\}|$?

The problem changes drastically when we consider a thickening version, i.e. when P is a set of unit balls and L be a set of tubes of radius 1.

We will survey some classical and modern incidence theorems and discuss their connection to combinatorics, geometric measure theory, harmonic analysis, and dynamics.

何旭华

Title: Towards a Geometric Theory of Characters

Abstract: In the field of representation theory, understanding the behavior of characters has long been a central pursuit. Characters are fundamental objects that encode crucial information about the symmetries inherent in mathematical structures. In this talk, we embark on an exciting journey towards a geometric theory of characters, a captivating framework that reveals hidden connections between algebraic geometry, combinatorics, and the vast landscape of representation theory.

We will begin by exploring the original algebraic definition of characters, focusing on their significance in groups and group algebras. From there, we will delve into Lusztig's theory of character sheaves on $GL_n(\bar{F}_q)$, which serves as a geometric counterpart to characters of the finite group $GL_n(F_q)$. This geometric perspective unveils remarkable connections between algebraic geometry of algebraic groups and their flag variety and the study of characters of finite groups of Lie type. In the end, we will embark on an ongoing project aimed at extending the theory of character sheaves to loop groups.

王博潼

Title: Perverse sheaves and positivity of Euler characteristics

Abstract: Some geometric or topological properties of a space have or conjecturally have implications on the sign of its Euler characteristics. For example, the Singer-Hopf conjecture predicts that if a closed $2d$ -dimensional manifold M has non-positive sectional curvature, or more generally has contractible universal cover, then $\chi(M) \geq 0$. We will discuss a few such examples and their generalizations in terms of perverse sheaves.

恽之玮

Title: Functions on the commuting scheme via Langlands duality

Abstract: I will explain how ideas from the (geometric) Langlands program help solve the following purely algebraic problem: describe the ring of conjugation-invariant functions on the scheme of commuting pairs in a complex reductive group. The answer was known up to nilpotents, and we show that this ring is indeed reduced. We also describe the ring of invariant functions on the derived version of the commuting scheme. The proof brings in seemingly unrelated objects such as the affine Hecke category and character sheaves (of the Langlands dual group).

This is joint work with Penghui Li and David Nadler.

申皓

Title: Stochastic quantization of Yang-Mills

Abstract: Stochastic quantization is a bridge between quantum field theory on one hand and stochastic analysis on the other hand. Through this connection, it is possible to put certain functional integrals in quantum field theory on rigorous footing, and prove various properties of them; I will discuss some recent progress and ongoing efforts. These studies also foster the developments of stochastic analysis so the two fields cross-fertilize. I will focus on our work with M.Hairer et al on the Yang-Mills model in 2 and 3 dimensions, discuss the construction of singular orbit space, local stochastic dynamics, singular holonomies etc. On lattice, stochastic quantization can be also used to prove interesting properties such as mass gap at strong coupling.





应用数学联系人：谭晓妮 62744132

基础数学联系人：余 萌 62744121