ASSOCIATED VARIETIES AND UNIPOTENT REPRESENTATIONS

1. Jeffrey Adams (Maryland)

Title. Unipotent Representations of Real Groups

Abstract. I will discuss the computation of unipotent representations of real exceptional groups.

2. Dan Barbasch (Cornell)

Title. Dirac Cohomology and Unitarity

Abstract. The classification of the unitary dual of a real reductive group is a central problem in the representation theory of such groups. The Dirac operator plays an important role in singling out an important subset of the unitary dual. For example, via the index theorem, work of Atiyah and Schmid classify the discrete series which are essential for the Plancherel formula. Unitary representations with nontrivial $(\mathfrak{g}, K)$-cohomology play an important role in the theory of automorphic forms. Dirac cohomology (introduced by Vogan) is another invariant which can be viewed as a generalization of $(\mathfrak{g}, K)$-cohomology. In this talk I will discuss results about the unitary representations with nontrivial Dirac cohomology. This is a report on joint work with Pavle Pandzic, Chao-Ping-Dong and Daniel Wong.

3. Chaoping Dong (Shanghai)

Title. Dirac series: Two infinities

Abstract. Dirac cohomology was an invariant for Lie group representations introduced by Vogan in 1997. In view of the research announcement of Barbasch and Pandzic in 2019, Dirac cohomology could be used to construct automorphic forms. In this talk, I will mention our works on the classification of the Dirac series. In particular, I will briefly talk about recent works on classical groups (complex and real), which are joint with Barbasch and Wong.

4. Xuhua He (CUHK)

Title. Flag varieties over semifields

Abstract. In 1994, Lusztig developed the theory of total positivity for arbitrary split real reductive groups and their flag manifolds. Later the theory has found important applications in different areas: cluster algebras, higher Teichmuller theory, the theory of amplituhedron in physics, etc.

Recently, Lusztig initiated the study of Kac-Moody monoids over arbitrary semifield and their flag manifolds. In the case where the Kac-Moody datum comes from a real reductive group and the semifield is $\mathbb{R}_{>0}$, the Kac-Moody

monoid over $\mathbb{R}_{>0}$ is exactly the totally nonnegative part of the real reductive group.

In this talk, I will discuss my joint work with Huanchen Bao on the flag manifolds $\mathcal{B}(K)$ over arbitrary semifield K and associated to any Kac-Moody datum G. We show that $\mathcal{B}(K)$ admits a natural action of the Kac-Moody monoid $G(K)$ and admits a decomposition into cells.

5. Jing-Song Huang (HKUST)

Title. The model SL_2- Fourier Transforms, Dirac Indices and Unipotent Representations

Abstract. A model SL_2 in a real reductive Lie group corresponds to a model nilpotent orbit (a maximal spherical nilpotent orbit). Here, a real orbit being spherical means that its complexified orbit is spherical, namely a Borel subgroup has an open orbit. The model SL_2 is related to

(1) the generalized Fourier transforms for real reductive groups.
(2) the Dirac indices of Harish-Chandra modules associated with the model orbits by Springer correspondence.
(3) the unipotent representations with multiplicity free K-types.

6. Dihua Jiang (Minnesota)

Title. Automorphic Descent and Related Problems.

Abstract. Automorphic descent is a construction method to construct concrete cuspidal automorphic modules using the global Arthur parameters. In this talk, I will discuss recent progress on the theory of automorphic descents and applications to related problems, which includes my work joint with Lei Zhang, Baiying Liu, and Bin Xu. If time permits, I will also mention some new development on the local theory, which is joint with Dongwen Liu and Lei Zhang.

7. Wen-wei Li (BICMR)

Title. On certain Ext-analogues of the distinction of Harish-Chandra modules

Abstract. Consider a connected reductive real group G and a spherical subgroup H. The $\mathfrak{h}$-invariant linear functionals on a Harish-Chandra module can be interpreted in terms of its localization, which is a D-module on G/H. I will sketch a proof that the localization, together with all its higher derived functors, are regular holonomic D-modules. Using this, I will try to explain how this can be applied to understand the Ext-analogues of the distinction of Harish-Chandra modules. It also leads to a new interpretation of the corresponding Euler-Poincaré indexes. This is a work in progress.

8. Jiajun Ma (Shanghai)

Title. Special unipotent representations of real classical groups and theta correspondence.

Abstract. In this talk, I will discuss a recent work (in progress) joint with Dan Barbasch, Binyong Sun, and Chengbo Zhu on the construction of special unipotent representations with integral infinitesimal characters of the real classical groups (real symplectic groups, real orthogonal groups, and the metaplectic groups). Special unipotent representations are certain irreducible admissible representations attached to special nilpotent orbits. They are expected to be unitarizable and to form the unipotent Arthur packet. Barbasch and Vogan established the theory of special unipotent representations for complex groups (construction, unitarizability, character formula, etc.).

Theta correspondence is an effective tool to construct singular representations of classical groups. Combining a counting method of unipotent representations and a formula on the associated characters of the theta liftings, we will show that, for a class of special nilpotent orbits (let us call them "strongly even"),

(1) iterated theta lifting constructs all special unipotent representations attached to these orbits;
(2) they are all unitarizable; and
(3) they are entirely determined by the associated character. The general cases are expected to reduce to the "strongly even" orbits via irreducible parabolic and cohomological inductions.

Some examples will be discussed through the talk.

9. Lucas Mason-Brown (Oxford)

Title. What is a unipotent representation (of a complex reductive group)?

Abstract. Let G be a connected reductive algebraic group, and let $G(\mathbb{F}_q)$ be its group of $\mathbb{F}_q$-rational points. Denote by $\mathrm{Irr}_{\mathrm{fd}}(G(\mathbb{F}_q))$ the set of (equivalence classes) of irreducible finite-dimensional representations. Deligne and Lusztig defined a finite subset $\mathrm{Unip}(G(\mathbb{F}_q)) \subset \mathrm{Irr}_{\mathrm{fd}}(G(\mathbb{F}_q))$ of unipotent representations. These representations play a distinguished role in the representation theory of $G(\mathbb{F}_q)$. In particular, the classification of $\mathrm{Irr}_{\mathrm{fd}}(G(\mathbb{F}_q))$ reduces to the classification of $\mathrm{Unip}(G(\mathbb{F}_q))$.

Now replace $\mathbb{F}_q$ with a local field k and replace $\mathrm{Irr}_{\mathrm{fd}}(G(\mathbb{F}_q))$ with $\mathrm{Irr}_u(G(k))$ (irreducible unitary representations). Vogan has predicted the existence of a finite subset $\mathrm{Unip}(G(k)) \subset \mathrm{Irr}_u(G(k))$ which completes the following analogy $\mathrm{Unip}(G(k))$ is to $\mathrm{Irr}_u(G(k))$ as $\mathrm{Unip}(G(\mathbb{F}_q))$ is to $\mathrm{Irr}_{\mathrm{fd}}(G(\mathbb{F}_q))$.

In this talk I will propose a definition of $\mathrm{Unip}(G(k))$ when $k = \mathbb{C}$. The definition is geometric and case-free. It is based on Ivan Loseu's work on quantizations of symplectic singularities. The representations considered include all of Arthur's, but also many others. After sketching the definition and cataloging its properties, I will explain a classification of $\mathrm{Unip}(G(\mathbb{C}))$, generalizing the well-known result of Barbasch-Vogan for Arthur's representations. Time permitting, I will discuss some speculations about the case of $k = \mathbb{R}$.

This talk is based on forthcoming joint work with Ivan Loseu and Dmitryo Matvieievskyi.

10. Stephen Miller (Rutgers)

Title. Eisenstein series and unitary representations

Abstract. I plan to give number-theoretic background on Eisenstein series and explain how (following a brilliant discovery by Birgit Speh from the 1980s) they have been used to prove certain representations are unitary, notably including many unipotent representations on exceptional groups such as E_8. Time permitting, I'll discuss some generalizations to Kac-Moody groups which suggest possible constructions of theta functions in the infinite-dimensional setting.

11. Yoshiki Oshima (Osaka)

Title. Restriction of unitary representations of $\mathrm{Spin}(N,1)$ to parabolic subgroups

Abstract. The orbit method predicts a relation between restrictions of unitary representations and projections of corresponding coadjoint orbits. In this talk we will discuss branching laws for unitary representations of $\mathrm{Spin}(N,1)$ restricted to parabolic subgroups and the corresponding orbit geometry. In particular, we confirm Duflo's conjecture in this setting. This is a joint work with Gang Liu (Lorraine) and Jun Yu (BICMR).

12. Annegret Paul (Western Michigan)

Title. Nilpotent Orbits in Atlas

Abstract. The Atlas software can be used for various calculations involving nilpotent orbits in real and complex Lie algebras, such as dimension, weighted diagrams, and other invariants. Of particular interest are centralizers and their component groups. I will give an overview of the capabilities and some of the mathematics behind the code, including specific commands and examples.

13. Pavle Pandzic (Zagreb)

Title. Dirac index and associated cycles of Harish-Chandra modules

Abstract. We show how, for certain Harish-Chandra modules, the polynomial giving the dimension of the Dirac index of the corresponding coherent family can be expressed as an integer linear combination of the coefficients of the characteristic cycle. This is joint work with S. Mehdi, D. Vogan and R. Zierau.

14. David Vogan (MIT)

Title. Tensor product structure on unipotent representations

Abstract. Suppose G is a complex reductive algebraic group, and I is a maximal two-sided ideal in the enveloping algebra $U(\mathfrak{g})$. We can consider the category $M(G,I)$= Harish-Chandra bimodules for $\mathfrak{g}$ annihilated by I. This category has the structure of a monoidal category, by $X * Y = X \otimes_{U(\mathfrak{g})} Y$. The problem of understanding the structure of this monoidal category in general is an interesting one, to which I don't know the answer.

Suppose now that I is one of the maximal ideals defined by Barbasch and myself in connection with Arthur's conjectures, attached to a nilpotent orbit $\mathcal{O}^\vee$ in the dual Lie algebra. We proved (at least in part) that the monoidal category in this case is that of finite-dimensional representations of the Lusztig's canonical quotient A of the fundamental group of $\mathcal{O}^\vee$. In particular, this means that each X in $M(G, I)$ defines a finite-dimensional representation $\xi(X)$ of A.

Suppose further that $G(\mathbb{R})$ is a real form of G, and that $\mathcal{O}^\vee$ comes from an Arthur parameter for $G(\mathbb{R})$. In this case Adams, Barbasch, and I attached to ψ a finite group $A(\psi)$ equipped with a homomorphism to A. We defined a category

$$M(G(\mathbb{R}), \psi) = \text{Harish-Chandra modules for } G(\mathbb{R}) \text{ annihilated by } I, \text{ and } ***$$

where the conditions $***$ mean in particular that each Z in $M(G(\mathbb{R}), \psi)$ defines a finite-dimensional representation $\tau(Z)$ of $A(\psi)$.

I will explain how to define an action of $M(G, I)$ on $M(G(\mathbb{R}), \psi)$, $X * Z = X \otimes|_{U(\mathfrak{g})} Z$, and explain why it seems likely that $\tau(X*Z) = (\xi(X)|A(\psi)) \otimes \tau(Z)$. Finally, I will explain how these ideas might help to prove Arthur's conjecture: unitarity of the representations in $M(G(\mathbb{R}), \psi)$.

15. Daniel Wong (S.Z. CUHK)

Title. Some special unipotent representations attached to spherical nilpotent orbits

Abstract. In this talk, we will study special unipotent representations for Hermitian symmetric groups with non-zero Dirac cohomology. It turns out that all such representations are attached to a spherical nilpotent orbit. We will investigate the structure of these representations, along with their possible relationships with the orbit method. This is a joint work with C.-P. Dong.

16. Shilin Yu (Xiamen)

Title. Deformation quantization of coadjoint orbits

Abstract. The coadjoint orbit method philosophy of Kirillov and Kostant suggests that irreducible unitary representations of a Lie group arise as quantizations of coadjoint orbits of the group. Later it was reformulated by Vogan in the case of real reductive Lie groups. We confirm partially Vogan's conjecture using deformation quantization. The talk is based on a joint paper with Conan Leung and an ongoing joint work with Ivan Losev.

17. Chengbo Zhu (Singapore)

Title. On the role of moment maps in local theta correspondence

Abstract. We consider two types of nilpotent invariants associated to smooth representations, namely generalized Whittaker models and associated characters. We will survey some recent results on the behavior of these nilpotent invariants under local theta correspondence, and highlight the special role of a certain double fibration of moment maps.

18. Yongchang Zhu (HKUST)

Title. Twisted Loop Groups and Their Weil Representations

Abstract. A twisted loop group of $\mathrm{SL}(n)$ can be viewed both as an orthogonal group and as a symplectic group. We view it as a symplectic group and construct its Weil representation and theta functionals. And we will discuss the theta lifting.