Quantitative methods for the mean field limit problem

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Abstract

We study the mean field limit of large systems of interacting particles. Classical mean field limit results require that the interaction kernels be essentially Lipschitz. To handle more singular interaction kernels is a long-standing and challenging question but which now has some successes. Joint with P.-E. Jabin, we use the relative entropy between the joint law of all particles and the tensorized law at the limit to quantify the convergence from the particle systems towards the macroscopic PDEs. This method requires to prove large deviations estimates for non-continuous potentials modified by the limiting law. But it leads to explicit convergence rates for all marginals. This in particular can be applied to the Biot-Savart law for 2D Navier-Stokes. To treat more general and singular kernels, joint with D. Bresch and P.-E. Jabin, we introduce the modulated free energy, combination of the relative entropy that we had previously developed and of the modulated energy introduced by S. Serfaty. This modulated free energy may be understood as introducing appropriate weights in the relative entropy to cancel the most singular terms involving the divergence of the kernels. Our modulated free energy allows to treat gradient flows with singular potentials which combine large smooth part, small attractive singular part and large repulsive singular part. As an example, a full rigorous derivation (with quantitative estimates) of some chemotaxis models, such as the Patlak-Keller-Segel system in the subcritical regimes, is obtained.