Erdös-Kac theorem revisited

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Abstract

Sample a random integer from $\{1, ..., n\}$ and consider the number $\omega(J_n)$ of distinct prime divisors of the sample J_n. Notice that ω is an additive function, i.e. $\omega(jk) = \omega(j) + \omega(k)$ whenever j, k are co-prime natural numbers. The asymptotic normality for the prime divisor counting function was remarkably established by Erdös and Kac, constituting the most fundamental result in probabilistic number theory. The optimal rate of convergence in Kolmogorov distance was later obtained by Rényi and Turán by complex-analytic methods which are based on sophisticated manipulations of the associated characteristic function. In this talk, we consider general additive functions and present a new probabilistic perspective. In order to show Gaussian fluctuation with optimal rates, we adapt ideas from Stein's method for normal approximation. In the case where the additive function of interest is the prime divisor counting function, we also show Poisson approximation with optimal rates in the total variation distance. This talk is based on a joint work with with Louis H.Y. Chen (Singapore) and Arturo Jaramillo (Luxembourg & Singapore).