Beijing-Zurich Moduli Workshop

September 9-12, 2019
Lecture Hall, Jiayibing Building, Jingchunyuan 82, BICMR, Peking University

Younghan Bae (ETH Zurich)

Tautological relations on the moduli space of stable maps

There is a notion of tautological relations on the moduli space of stable maps to a target variety. One way to obtain universal tautological relations is to use the double ramification cycles on stable map spaces. We will discuss various methods to obtain tautological relations.

Pierrick Bousseau (ETH Zurich)

Scattering diagrams, sheaves on $\mathbb{P}^2$ and curves in $\mathbb{P}^2$ relative to a smooth cubic

I will explain how the algebraic notion of scattering diagram can be used to unify two a priori distinct topics: the wall-crossing behavior of moduli spaces of Bridgeland semistable objects in the derived category of coherent sheaves on $\mathbb{P}^2$, and Gromov-Witten theory of $\mathbb{P}^2$ with maximal tangency at a point along a given smooth cubic. Once this connection established, I will explain how knowledge on the sheaf side gives us a way to solve questions on the Gromov-Witten side, such as N. Takahashi's conjecture on the dependence on the point of contact with the cubic.

Tim-Henrik Bülles (ETH Zurich)

On the Invariance Property Conjecture for $K3$ surfaces

I will discuss how the monodromy over the moduli space of $K3$ surfaces imposes constraints on the Gromov-Witten classes. The discussion is motivated by the Invariance Property Conjecture for GW invariants of $K3$ surfaces.

Honglu Fan, Longting Wu (ETH Zurich), and Fenglong You (University of Alberta)

Relative and orbifold Gromov-Witten theory

In the series of talks, we will study the structures of relative Gromov-Witten (GW-) theory through orbifold GW-theory. In the first talk, Fenglong You will show the connection between relative GW-theory and orbifold GW-theory of the root stack. In the second talk, by generalizing the definition of relative GW-theory to include negative contact orders, Honglu Fan will introduce the quantum cohomology for relative GW-theory. The idea to introduce negative contact orders is very natural from the orbifold point of view. In the third talk, Longting Wu will show that the WDVV equation (or equivalently, the associativity of our relative quantum cohomology ring) can be used to calculate certain relative GW-invariants. This is based on joint work with Hsian-Hua Tseng.

Tom Graber (Caltech)

Torus localization for relative obstruction theories and log stable maps

I will describe a localization formula for the virtual fundamental class associated to a relative perfect obstruction theory over a possibly singular base. The motivating case is the space of log stable maps to a toric variety.

Zhiyuan Li (Fudan University)

Arithmetic period map and Shafarevich type questions for HK varieties

Let K be a number field and S be a finite set of places of K. Shafarevich made the question about the finiteness of the set of isomorphism classes of varieties over K with good reduction outside of S. Faltings and André have proved this conjecture for curves of fixed genus, polarized abelian surfaces and HK varieties. In this talk, I will talk about the unpolarized Shafarevich type questions and explain how they relate to the Torelli theorem over number fields. This is an ongoing project with Lie Fu.

Georg Oberdieck (University of Bonn)

Gromov-Witten theory of $T^*E \times \mathbb{P}^1$

I will report on a joint project with Aaron Pixton in which we determine the Gromov-Witten theory of the product of the cotangent bundle of an elliptic curve with the projective line, relative to two fibers over the projective line. The matrix elements are the Fourier expansions of Jacobi forms.

An action of the Verbitsky algebra on Chow

We construct an action of the Verbitsky Lie algebra on the Chow ring of the Hilbert scheme of points of a $K3$ surface. This yields another argument for Beauville's conjecture (first proven by Maulik and Neguţ) that the cycle class map is injective on the subring generated by divisor classes. In the second half I will explain some applications to holomorphic anomaly equations for Hilbert schemes.

Rahul Pandharipande (ETH Zurich)

Double ramification cycles, DR(X), and log DR

A basic question in the theory of algebraic curves is whether a divisor represents the zeros and poles of a rational function. An explicit solution in terms of periods was given by the work of Abel and Jacobi in the 19th century. In the past few years, a different approach to the question has been pursued: what is the class in the moduli of pointed curves of the locus of such divisors? I will explain the answer in Gromov-Witten theory given by Pixton's formula for the double ramification cycle. More recently, the subject has been developing in several new directions which I will present: DR cycles with targets, loci of meromorphic differentials, and work in progress on the logarithmic case.

Yongbin Ruan (University of Michigan)

Verlinde/Grassmannian correspondence and quantum K-theory

More than twenty years ago, Witten proposed an equivalence of two quantum fields governing Verlinde algebra (or the theory of stable bundles over a curve) and the quantum cohomology of Grassmannian. Motivated by Witten's physical work and recent revival of quantum K-theory, we proposed a K-theoretic version of so-called Verlinde/Grassmannian correspondence. Furthermore, the recent interpretation of quantum K-theory as a 3d quantum field theory opens a door to much larger area of physics and mathematics. We will first review the new ingredient of level structure in quantum K-theory and surprising appearance of mock theta function. Then, we will present an approach to the proof of correspondence using wall-crossing technique. This is a joint work with Ming Zhang.

Qizheng Yin (Peking University)

Topology of Lagrangian fibrations and Gopakumar-Vafa invariants of $K3$ surfaces

I will present a new connection between the topology of (holomorphic) Lagrangian fibrations and the Hodge theory of compact hyper-Kähler manifolds. Lagrangian fibrations induced by the Beauville-Mukai system are closely related to the enumerative geometry of $K3$ surfaces. One consequence of this is a new mathematical proof that the (refined) Gopakumar-Vafa invariants of $K3$ surfaces depend only on the genus and the degree of the curve class, not on the divisibility of the curve class. Joint work with Junliang Shen.