KLOOSTERMAN CRYSTALS FOR REDUCTIVE GROUPS

I will first review the relationship between the classical Bessel equation

$$(x\frac{d}{dx})^2 u - xu = 0,$$

and the classical Kloosterman sum

$$\mathrm{Kl}(a) := \sum_{xy=a\in\mathbb{F}_p} \exp(\frac{2\pi i}{p}(x+y)).$$

Such a relation can be regarded as an instance of the geometric Langlands correspondence for GL_2. I will survey the recent generalizations of this story for arbitrary reductive groups, based on the works by Frenkel-Gross, Heinloth-Ngô-Yun, and X. Zhu. In the end, I will report the joint work in progress with X. Zhu, where we study the p-adic aspect of this theory.