# Conference for the 10th Anniversary of the Enhanced Program for Graduate Study

Peking University, Beijing, China May 24-25, 2019



Supported by Beijing International Center for Mathematical Research (BICMR)

### Speakers:

Xingshan Cui (Virginia Tech) Song Dai (Tianjin University) Changguang Dong (University of Maryland) Ke Feng (Peking University) Shaoming Guo (University of Wisconsin-Madison) Jialiang He (Bar-Ilan University) Kuijie Li (Fudan University) Caihua Luo (Chalmers University of Technology) Fei Qi (Yale University) Qiang Xu (Max Planck Institute,MPI) Ruobing Zhang (Stony Brook University)

# **Organization Committee:**

Xiaobo Liu (BICMR, Peking University) Zhaojun Sun (BICMR, Peking University)

### **Conference venue:**

Lecture Hall, Jiayibing Building, Jingchunyuan 82, BICMR (北京大学镜春园 82 号甲乙丙楼报告厅)

# Hotel:

FX Hotel ZhongGuanCun (富驿时尚酒店中关村店).

Address: No.68 North 4th Ring West Road (South of Haidian Bridge) Haidian District, Beijing 100080 China.



Here is some tips on how to arrive to BICMR from Airport and Railway station: http://bicmr.pku.edu.cn/content/page/9.html

Conference for the 10th Anniversary of the Enhanced Program for Graduate Study		
May24-25,2019 Lecture Hall, Jiayibing Building, Jingchunyuan 82, BICMR, Peking Universtiy		
Time	Friday May 24	Saturday May 25
8:40-9:10	Opening Remarks & Group Photo	
	Chair:Emanuel Scheidegger	Chair:Wenwei Li
9:10-10:00	Fei Qi (Yale University)	Changguang Dong (University of Maryland)
9:50-10:10	Break	Break
10:10-11:00	Xingshan Cui (Virginia Tech)	Shaoming Guo (University of Wisconsin-Madison)
11:00-11:10	Break	Break
11:10-12:00	Kuijie Li (Fudan University)	Jialiang He (Bar-Ilan University)
12:00-14:00	Lunch	Lunch
	Chair:Maozheng Guo	Chair:Huabin Ge
14:00-14:50	Song Dai (Tianjin University)	Ruobing Zhang (Stony Brook University)
14:50-15:00	Break	Break
15:00-15:50	Qiang Xu (Max Planck Institute,MPI)	Ke Feng (Peking University)
15:50-16:00	Break	Break
16:00-16:50	Caihua Luo (Chalmers University of Technology)	Free Discussion
17:30-19:30	Dinner in Yiyuan (艺园)	Dinner in Yiyuan (艺园)

# Title and abstracts:

### Xingshan Cui (Virginia Tech)

Title: Mathematics of Topological Quantum Computing

### Abstract:

Topological quantum computing (TQC) is among the best approaches to building a large-scale fault-tolerant quantum computer. The quantum media for TQC are topological phases of matter that harbor non-Abelian anyons and quantum gates are implemented by braiding of anyons. The mathematics of topological phases of matter is described by modular tensor categories or equivalently by topological quantum field theories. We give a review on the rich interactions between TQC and the subjects mentioned above. We illustrate the concept of TQC with an important class of anyons, namely, metaplectic anyons, and show that braidings of anyons assisted by certain topologically protected measurements is universal for quantum computing. The interest in metaplectic anyons arises from the potential physical realization in fractional quantum Hall systems. Time permitting, we also talk about the application of topological quantum field theories in topology and give a new invariant of smooth 4-manifolds of state-sum type.

### Song Dai (Tianjin University)

Title: Dominations in Higher Teichmüller Theory

### Abstract:

In this talk, we will survey some domination results in higher Teichmüller theory. Higher Teichmüller theory, initiated by Hitchin, studies the moduli space of the representations from a surface group to a semisimple Lie group, which is a generalization of classical Teichmüller theory. By the non-Abelian Hodge theory, the moduli space of the representations corresponds to the moduli space of the Higgs bundles via the equivariant harmonic maps from the universal cover of the Riemann surface to the symmetric space of the Lie group. We focus on the geometric quantities of the harmonic map, for instance the metric and the curvature. Under the background of the Hitchin fibration, we describe a conjectural picture for the behavior of these quantities and show some developments in recent years.

### Changguang Dong (University of Maryland College Park)

**Title**: An Introduction to Dynamical Systems and Rigidity Theory, with Applications to Other Fields

### Abstract:

We will first present an introduction to dynamical systems, and its connection to other fields in mathematics. Then we focus on rigidity theory and its applications to other fields such as number theory. We will give several attractive application examples.

# Ke Feng (Peking University)

Title: An Introduction to Monge-Ampère Equation in Geometric Analysis

# Abstract:

This talk is the preliminary introduction to Monge-Ampère equation. I will show some classical regularity results and applications, like existential problem for extremal metric in geometric analysis.

# Shaoming Guo (University of Wisconsin Madison)

**Title**: Decoupling Inequalities in Harmonic Analysis and Applications to Analytic Number Theory

# Abstract:

I will present a few results in decoupling theory that are related to analytic number theory. These results are related to counting integral solutions to Vinogradov system, Parsell-Vinogradov systems and Arkhipov-Chubarikov-Karatsuba systems. Applications to Waring's problem and its higher dimensional generalisations will also be discussed.

Jialiang He (Bar-Ilan university) Title: An Introduction to Selection Principle Mathematics Abstract: A selection principle is a rule asserting the possibility of obtaining mathematically significant objects by selecting elements from given sequences of sets. The theory of selection principles studies these principles and their relations to other mathematical properties. Selection principles mainly describe covering properties, measure- and category-theoretic properties, and local properties in topological spaces, especially function spaces. Often, the characterization of a mathematical property using a selection principle is a nontrivial task leading to new insights on the characterized property.

In this talk, I will introduce some basic concepts of selection principle mathematics and connections with other mathematics branches.

#### Kuijie Li (Fudan University)

Title: Navier-Stokes Equations: Existence and Regularity Criteria

#### Abstract:

There has been extensive literature on the study of incompressible Navier-Stokes(NS) equations. In this talk, i first recall some preliminary results on the local existence and uniqueness of the solutions in critical spaces, which date back to Kato. The global existence problem seems out of reach for the moment, however, under additional assumptions, we can obtain global regularity. These are now known as regularity criteria and i will review parts of them. At the end, a regularity criterion for NS equations in higher dimensions will be present. This is a joint work with Prof. Wang.

Caihua Luo (Chalmers University of Technology and the University of Gothenburg)

### Title: Langlands Program: the Local Aspect

### Abstract:

Around 1970s, Langlands proposed a clutch of far-reaching and influential conjectures to make a bridge between number theory and geometry. Roughly speaking, it seeks to relate Galois group in algebraic number theory to automorphic forms and representation theory of algebraic groups over local fields and adeles. This talk is about the representation theory of algebraic groups over local fields, especially the

longstanding problem on the structure of parabolic inductions following Harish-Chandra's "Philosophy of cusp forms".

### Fei Qi (Yale University)

Title: Cohomology Theory of Meromorphic Open String Vertex Algebras

### Abstract:

In the study of associative algebras and modules, cohomology theory plays an important role. In particular, the first Hochschild cohomology is a vector space isomorphic to the space of outer derivations; if the first cohomology vanishes for every bimodule, then every left module for the algebra is completely reducible; and the second Hochschild cohomology is in one-to-one correspondence with the set of first-order deformations of the algebra. We would like to develop cohomology methods for vertex algebras. A vertex algebra is the algebraic structure formed by vertex operators, which are suitable infinite series of endomorphisms on a vector space. These vertex operators satisfy conditions that are analogous to those in a commutative associative algebra. Yi-Zhi Huang discovered in 2012 that the cohomology theory can also be defined for grading-restricted vertex algebras and obtained analogues of some of the previously mentioned results. Joint with Huang, the speaker generalized the theory for meromorphic open-string vertex algebras (where commutativity does not hold and can be viewed as analogues of noncommutative associative algebras) that are not necessarily grading-restricted and obtained analogues of all the previously mentioned results.

#### Qiang Xu (Max Planck Institute)

### Title: An Introduction to Stochastic Homogenization Theory

#### Abstract:

In this talk, we start from periodic setting to overview some basic knowledge on homogenization theory, such as corrector and two-scale expansion methods. Then we move to a qualitative result in stochastic homogenization problems, and the connection between the sub-linear growth property of the corrector and homogenization phenomenon would be revealed there. Moreover, under a qualification of ergodicity we obtain some quantitative results, such as quenched and annealed Calderón-Zygmund theory, as well as systematic error estimates. Throughout the talk, we focus on linear elliptic equations as the model.

### Ruobing Zhang (Stony Brook University)

**Title**: Quantitative Geometric Structures, Regularity Theory and New Constructions of Collapsed Einstein Spaces

#### Abstract:

An Einstein metric, by definition, is a Riemannian metric with constant Ricci curvature. Roughly, the metric satisfies some highly degenerate nonlinear elliptic system. This talk centers on the degenerations or limiting behaviors of a family of Einstein metrics on a fixed or family of underlying manifolds, which is an active interdisciplinary research topic in the very general context of differential geometry such as Riemannian geometry and complex differential geometry. We are interested in families of Einstein manifolds which limit to lower dimensional spaces such that sufficiently wild analytic properties naturally arise. We will explain some new tools and various points of view in analyzing Einstein equations. Specifically, our crucial technical point is to dominate the degeneration geometries at some uniform and rather canonical scales. Such flavor of quantitative analysis will lead to new regularity theorems and new constructions of collapsed Einstein spaces.