INVARIANT THEORY: CLASSICAL, QUANTUM AND SUPER.

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Overview

This will be a series of 14 hours of lectures (7 lectures of 2 hours each) given at the University of Peking in May 2019 whose unifying theme is invariant theory.

Some of the most important intractable problems in algebra and geometry may be expressed in terms of the decomposition of tensor products of well-understood modules. For example the decomposition of $V^{\otimes r}$, where $V = \mathbb{F}_p^n$, regarded as a module for $\mathrm{GL}_n(\mathbb{F}_p)$ where p is a prime, includes the problems of determining the dimensions of the simple $\mathbb{F}_p$-modules for both Sym_n and $\mathrm{GL}_n(\mathbb{F}_p)$. These are both difficult and important open problems.

The course will include the fundamental theorems of invariant theory, as well as geometric and diagrammatic methods. Many situations discussed will be in a semi-simple context, with generalisation to the non-semi-simple case achieved via cellular deformation.

About the content

A selection of the following topics will be covered.

- Semi-simple modules and algebras.
- Hopf algebras and tensor representations.
- Three formulations of the fundamental theorems of classical invariant theory.
- Proof of the fundamental theorems for $\mathrm{GL}_n(\mathbb{C})$ and $V = \mathbb{C}^n$.
- Elementary notions of algebraic geometry.
- Algebro-geometric proof of the fundamental theorems for $\mathrm{O}_n(\mathbb{C})$ and $\mathrm{Sp}_{2n}(\mathbb{C})$.
- R-matrices and representations of the braid group on $W^{\otimes r}$ where W is a module for the quantum group $\mathrm{U}_q(\mathfrak{g})$.
- Quantum versions of the fundamental theorems.
- Non-semi-simple deformations.
- Cellular algebras and cellular categories.
- The Temperley-Lieb category and $\mathrm{U}_q(\mathfrak{sl}_2)$.
- Tangle categories and diagram algebras.
- Lie superalgebras, e.g. $\mathfrak{osp}(m|2n)$ and their representation theory.

- Some theorems for roots of unity.
- Infinite dimensional representations and polar tangle categories.

References

[1] Andersen, Henning H.; Lehrer, Gustav I.; Zhang, Ruibin, " Cellularity of certain quantum endomorphism algebras", *Pacific J. Math.* **279** (2015), no. 1-2, 11–35.

[2] Deligne, P.; Lehrer, G. I.; Zhang, R. B., " The first fundamental theorem of invariant theory for the orthosymplectic super group", *Adv. Math.* **327** (2018), 4–24.

[3] Kenji Iohara, Gus Lehrer, Ruibin Zhang, "Schur-Weyl duality for certain infinite dimensional Uq(sl2)-modules ", arXiv:1811.01325 .

[4] Lehrer, G. I.; Zhang, R. B. "Strongly multiplicity free modules for Lie algebras and quantum groups", *J. Algebra* **306** (2006), no. 1, 138–174.

[5] Lehrer, Gustav Isaac; Zhang, Ruibin, " On endomorphisms of quantum tensor space", *Lett. Math. Phys.* **86** (2008), no. 2-3, 209–227.

[6] Lehrer, G. I.; Zhang, R. B., " The Brauer category and invariant theory", *J. Eur. Math. Soc. (JEMS)* **17** (2015), no. 9, 2311–2351.

[7] Lehrer, Gustav I.; Zhang, Ruibin, " Invariants of the special orthogonal group and an enhanced Brauer category", *Enseign. Math.* **63** (2017), no. 1-2, 181–200.