“Eisenstein ideals and cuspidal subgroups of Jacobians of modular curves”
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Let N be a positive integer and let X be the classical modular curve $X_0(N)$. Consider the set of *cusps* on X. The cuspidal group attached to X is the image C in the Jacobian of X of the group of degree-0 divisors on X that are supported on the set of cusps of X. A theorem of Manin–Drinfeld asserts that C is finite. There is a great deal of literature to the effect that C is “large” in some qualitative sense. For example, view C as the quotient of the group of degree-0 cuspidal divisors on X by the group of divisors of the “modular units” on X. Theorems of Kubert and Kubert–Lang identify the group of modular units completely in terms of units that were constructed earlier by Siegel. Because all modular units may be described in terms of previously known units, it is natural to say that there are not too many of them. Qualitatively, to say that the group of modular units is small is to say that C is large.

Our theorem, proved in joint work with B. Jordan and A. Scholl, states that C is large in the sense that its annihilator in a ring of Hecke operators is as small as possible: namely, the annihilator is the Eisenstein ideal of the ring. We don’t prove this statement completely, but do prove it locally at all prime numbers ≥ 5 that are prime to N. As an application, we determine the structure of C as a Hecke module when N is square free, again locally at primes that do not divide $6N$.