Accuracy v.s. Implementability in Algorithmic Design —- An Example of Operator Splitting Methods for Convex Optimization

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Outline

1. Backgrounds
2. Accuracy v.s. Implementability – An Easier Case
3. Accuracy v.s. Implementability – A More Complicated Case
4. Conclusions

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- Yet, they are usually conflicted (to be proved later).

A Canonical Convex Optimization Model

- A canonical convex minimization model with linear constraints:

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  which solves an easier problem without linear constraints — with much more implementability.
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- With sufficient implementability while too little accuracy.

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- It is convergent with any fixed $\beta > 0$ (higher accuracy)

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- It can be regarded as a dual ascent method over the dual variable λ.
- A significant difference from the penalty method — the penalty parameter of ALM can theoretically be fixed as any positive scalar.

Outline

A Separable Model

- For many applications, the last model can be specified as a separable form

$$\min\{\theta_1(x_1) + \theta_2(x_2) \mid A_1x_1 + A_2x_2 = b,\ x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2\},$$

where $A_1 \in \Re^{m\times n_1}$, $A_2 \in \Re^{m\times n_2}$, $b \in \Re^m$, $\mathcal{X}_i \subseteq \Re^{n_i}$ $(i = 1, 2)$ and $\theta_i : \Re^{n_i} \to \Re$ $(i = 1, 2)$.

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- A typical application of the widely-used l_1-l_2 model

$$\min\{\mu\|x\|_1+\frac{1}{2}\|Ax-b\|^2\}$$

  where the least-square term $\frac{1}{2}\|Ax-b\|^2$ represents a data-fidelity term and the l_1-norm term $\|x\|_1$ is a regularization term for inducing spare solutions, and $\mu>0$ is a trade-off parameter.

Using ALM Directly with 100% Accuracy

Applying ALM directly:

$$\begin{cases} (x_1^{k+1}, x_2^{k+1}) = \arg\min\{\theta_1(x_1) + \theta_2(x_2) - (\lambda^k)^T(A_1x_1 + A_2x_2 - b) + \frac{\beta}{2}\|A_1x_1 + A_2x_2 - b\|^2 \mid (x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2\} \\ \lambda^{k+1} = \lambda^k - \beta(A_1x^{k+1} + A_2x_2^{k+1} - b); \end{cases}$$

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Is it easy to solve the ALM subproblem exactly?

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- Sequential Splitting is convergent — the Alternating Direction Method of Multipliers (ADMM) originally proposed by R. Glowinski and Marrocco in 1975.

Comments on ADMM

The ADMM scheme:

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- For the mentioned l_1-l_2 model, all subproblems are even easy enough to have closed-form solutions (to be delineated).

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- Review papers: Boyd *et al.* 2010, Glowinski 2012, Eckstein and Yao 2012.

Accuracy of ADMM

Certainly, acquiring implementability does not mean no care about the accuracy.

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- How to define "≈" rigorously above?
- For a general case, we need to analyze rigorously the inexactness criterion for solving these subproblems [1].

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Two ADMM Applications

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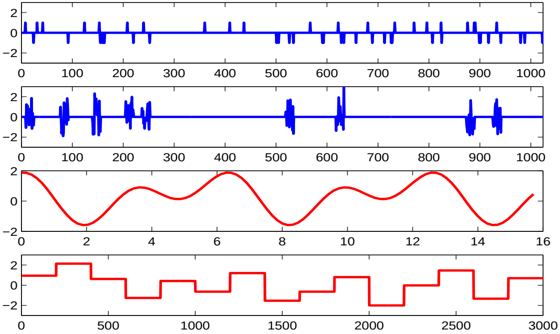
Ideal model: $Ax = b$

x — original signal, A — sensing matrix (a fat matrix), b — observation (with noise)

The Sparsity of a Signal

Some signals are large-scale but sparse (maybe under some transform domain)

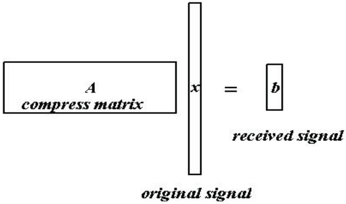


Mathematical Model

Find a sparse solution of a system of linear equations

$$\min \ \left\{ \|x\|_0 \ \middle| \ Ax = b, \ x \in \mathcal{R}^n \right\},$$

where $\|x\|_0 =$ number of nonzeros of x and $A \in \mathcal{R}^{m \times n}$ with $m \ll n$.



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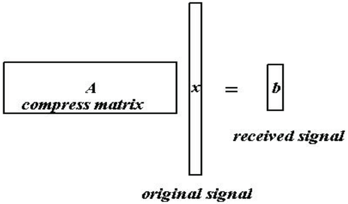

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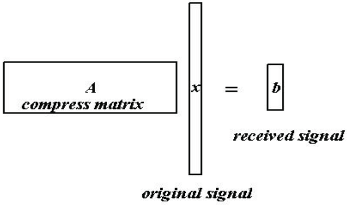

- The solution is in general not unique.
- It is NP-hard!

Basic Models for Compressive Sensing

- Basis-pursuit (BP):

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- l_1-regularized least-squares model:

$$\min \tau \|x\|_1 + \frac{1}{2}\|Ax - b\|_2^2$$

A Reformulation of the $l^1 - l^2$ Model

$$\min_x \ \tau\|x\|_1 + \frac{1}{2}\|Ax - b\|_2^2$$

$$\Updownarrow \ \text{By introducing } y$$

$$\begin{array}{ll} \min & \tau\|x\|_1 + \frac{1}{2}\|Ay - b\|_2^2 \\ \text{s.t.} & x = y. \end{array}$$

Solutions of ADMM's Subproblems

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P1 is a soft-shrinkage operator

P2 is a system of linear equations, efficient solvers (e.g. PCG or BB) are available

Another ADMM Application

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A clean image could be degraded by blur — defocus of the camera's lens, the moving object, turbulence in the air, $\cdots$

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$$\min \||\nabla x|\|_1 + \frac{\mu}{2}\|Kx - x^0\|^2,$$

where x is the clean image, x^0 is the corrupted image by Gaussian noise, K is the point spread function (blur), ∇ is a gradient operator (by Rudin/Osher/Fatemi, 92') to preserve sharp edges of an image, and μ is a trade-off parameter.

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original image

blurred image

restored image

Applying ADMM

Reformulate it as

$$\begin{aligned} \min \; & \| |y| \|_1 + \frac{\mu}{2} \|Kx - x^0\|^2 \\ \text{s.t. } & \nabla x = y, \end{aligned}$$

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- The x-subproblem (via a DFT):

$$\tilde{x}^k = \arg\min_x \left\{ \frac{\mu}{2}\|Kx - x^0\|^2 - (\lambda^k)^T(\nabla x - y^k) + \frac{\beta}{2}\|\nabla x - y^k\|^2 \right\}.$$

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$$\tilde{y}^k = \arg\min_y \left\{ |||y|||_1 - (\lambda^{k+1})^T(\nabla x^{k+1} - y) + \frac{\beta}{2}\|\nabla x^{k+1} - y\|^2 \right\}.$$

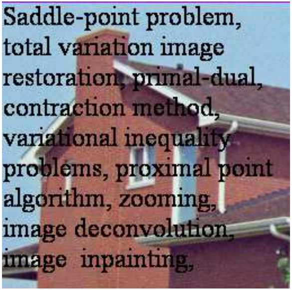
Image Inpainting

Problem: Some pixels are missing in image. Partial information of image is available

$\mathbf{g} = S\,\mathbf{f}, \quad S$ — mask

Model: $\min \ \{\|\nabla \mathbf{f}\|_1 \ | \ S\,\mathbf{f} = \mathbf{g}\}$

original image



missing pixel image

restored image

Image Decomposition

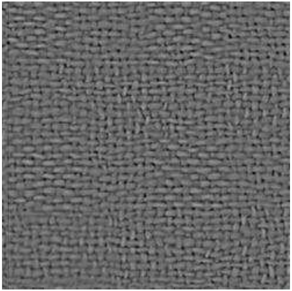
Problem: Separate the sketch (cartoon) and oscillating component (texture) of image

$\mathbf{f} = \mathbf{u} + \mathbf{v}$, $\mathbf{u}$ — cartoon part, $\mathbf{v}$ — texture part

Model: $\min \left\{ \tau \|\nabla \mathbf{u}\|_1 + \|\mathbf{v}\|_{-1,\infty} \;\middle|\; \mathbf{u} + \mathbf{v} = \mathbf{f} \right\}$

original image

cartoon part



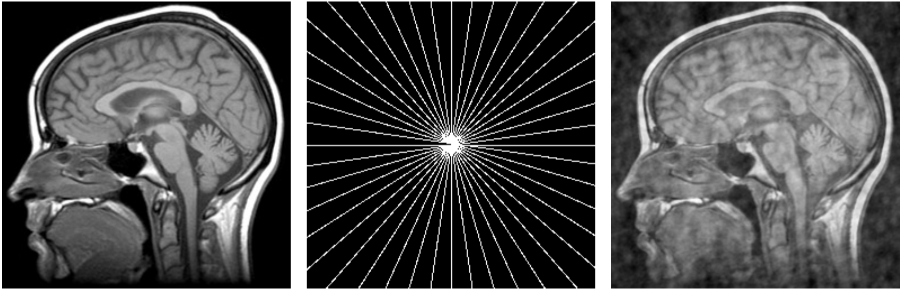
texture part

Magnetic Resonance Imaging (MRI)

Problem: Reconstruct a medical image by sampling its Fourier coefficients partially

$\mathcal{F}\mathbf{g} = P\mathcal{F}\mathbf{f}$, P — sampling mask, $\mathcal{F}$ — Fourier transform

Model: min $\{\|\nabla \mathbf{f}\|_1 \mid \mathcal{F}\mathbf{g} = P\mathcal{F}\mathbf{f}\}$



medical image sampling mask reconstruction

Outline

A More Complicated Model with Higher Degree of Separability

A more complicated multi-block separable convex optimization model:

$$\min \left\{ \sum_{i=1}^{m} \theta_i(x_i) \;\middle|\; \sum_{i=1}^{m} A_i x_i = b,\; x_i \in \mathcal{X}_i,\; i = 1, 2, \cdots, m \right\},$$

with $m \geq 3$.

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A more complicated multi-block separable convex optimization model:

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with $m \geq 3$.

Applications include

- Image alignment problem
- The robust principal component analysis model with noisy and incomplete data
- The latent variable Gaussian graphical model selection
- The quadratic discriminant analysis model

Splitting Versions with Less Accuracy while More Implementability

- Obviously, the parallel (Jacobian) splitting:

$$\begin{cases} x_1^{k+1} = \operatorname{argmin}\{\theta_1(x_1) - (\lambda^k)T(A_1x_1) + \frac{\beta}{2}\|A_1x_1 + \sum_{j=2}^{m} A_jx_j^k - b\|^2 \mid x_1 \in \mathcal{X}_1\}, \\ \cdots\cdots \\ x_i^{k+1} = \operatorname{argmin}\{\theta_i(x_i) - (\lambda^k)^T(A_ix_i) + \frac{\beta}{2}\| \sum_{j=1}^{i-1} A_jx_j^k + A_ix_i + \sum_{j=i+1}^{m} A_jx_j^k - b\|^2 \mid x_i \in \mathcal{X}_i\}, \\ \cdots\cdots \\ x_m^{k+1} = \operatorname{argmin}\{\theta_m(x_m) - (\lambda^k)^T(A_mx_m) + \frac{\beta}{2}\| \sum_{j=1}^{m-1} A_jx_j^k + A_mx_m - b\|^2 \mid x_m \in \mathcal{X}_m\}, \\ \lambda^{k+1} = \lambda^k - \beta(\sum_{i=1}^{m} A_ix_i^{k+1} - b). \end{cases}$$

does not work (more details are coming).

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- This direct extension of the ADMM has been widely used in the literature; and it does work very well for many applications!
- But for a very long time, neither affirmative convergence proof nor counter example showing its divergence was available.

- Recently we [2] found some examples showing the divergence of the direct extension of ADMM even when $m = 3$. So, the direct extension of ADMM for multi-block separable convex optimization model is not necessarily convergent!

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the following scheme is not necessarily convergent:

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[2] Chen/He/Ye/Y., The direct extension of ADMM for multi-block separable convex minimization models is not necessarily convergent, September 2013.

- Recently we [2] found some examples showing the divergence of the direct extension of ADMM even when $m = 3$. So, the direct extension of ADMM for multi-block separable convex optimization model is not necessarily convergent!
- That is, even to solve

$$\min \left\{ \theta_1(x_1) + \theta_2(x_2) + \theta_3(x_3) \;\middle|\; A_1x_1 + A_2x_2 + A_3x_3 = b,\; x_i \in \mathcal{X}_i,\; i = 1, 2, 3 \right\},$$

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- Both Jacobian and Gauss-Seidel decompositions fail — too much loss of accuracy for $m \geq 3$!

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One Way of Applying the ADMM

- Conceptually, we can treat the multi-block model as a two-block model

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- Then, apply the original ADMM (for the two-block case)

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- It is accurate (recall ADMM's convergence).
- But it is not implementable (hard to solve the (x_2,x_3)-subproblem).

ADMM with Further Splitting

- Split the (x_2, x_3)-subproblem in parallel

$$\begin{cases} x_1^{k+1} = \arg\min\{\theta_1(x_1) - (\lambda^k)^T(A_1x_1) + \frac{\beta}{2}\|A_1x_1 + A_2x_2^k + A_3x_3^k - b\|^2 \mid x_1 \in \mathcal{X}_1\}, \\ x_2^{k+1} = \arg\min\{\theta_2(x_2) - (\lambda^k)^T(A_2x_2 + A_3x_3^k - b) + \frac{\beta}{2}\|A_1x_1^{k+1} + A_2x_2 + A_3x_3^k - b\|^2 \mid x_2 \in \mathcal{X}_2\}, \\ x_3^{k+1} = \arg\min\{\theta_3(x_3) - (\lambda^k)^T(A_2x_2^k + A_3x_3 - b) + \frac{\beta}{2}\|A_1x_1^{k+1} + A_2x_2^k + A_3x_3 - b\|^2 \mid x_3 \in \mathcal{X}_3\}, \\ \lambda^{k+1} = \lambda^k - \alpha\beta(A_1x_1^{k+1} + A_2x_2^{k+1} + A_3x_3^{k+1} - b). \end{cases}$$

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- Proximally regularized the decomposed subproblems (this works even when the ALM subproblem is decomposed in parallel), see He/Xu/Y., Deng/Lai/Pang/Yin, Wang/Hong/Ma/Luo, etc.

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- Application-driven optimization makes sense!

Thank you!

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