Title & Abstract

Reduced order modeling techniques for multiscale methods

Assyr Abdulle (EPFL, Switzerland)

In this talk we discuss the use of reduced order modeling techniques such as the reduced basis in the design of multiscale methods for partial differential equations with highly oscillatory coefficients. Applications to quasilinear elliptic and parabolic problems and Stokes problems with multiples scales will be presented.

This talk is based upon a series of joint works with various collaborators (see the references below).

References

[1] A. Abdulle, Y. Bai, Reduced-order modelling numerical homogenization, Philos. Trans. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. 372 (2014), 20130388, 23 pp.
[2] A. Abdulle, Y. Bai, G. Vilmart, Reduced basis finite element heterogeneous multiscale method for quasilinear elliptic homogenization problems. Discrete Contin. Dyn. Syst., 8 (2015), pp. 91–118.
[3] A. Abdulle, O. Budac, An adaptive finite element heterogeneous multiscale method for Stokes flow in porous media, SIAM, Multiscale Model. Simul., 13 (2015), pp. 256–290.
[4] A. Abdulle, O. Budac, A Petrov-Galerkin reduced basis approximation of the Stokes equation in parametrized geometries, C. R. Math. Acad. Sci. Paris, 353 (2015), pp. 641–645.
[5] A. Abdulle, P. Henning, A reduced basis localized orthogonal decomposition, J. Comput. Phys., 295 (2015), pp. 379–401.
[6] A. Abdulle, O. Budac, A reduced basis finite element heterogeneous multiscale method for Stokes flow in porous media, (2015), preprint

First-order primal – dual algorithms for convex optimization in imaging science

Tony Chan (HKUST)，Joint work with Dr. Mingqiang ZHU

Recently a great amount of work has been done in developing first-order primal-dual algorithms to solve convex optimization problems in imaging. These algorithms are simple but efficient and thus has become state-of-the-art popular algorithms in practical imaging research.   In this talk, I will give a brief survey on recent works and developments in this area.

Offline and online adaptivity for a class of multiscale problems

Eric Chung (The Chinese University of Hong Kong)

Many real-world problems involve multiple scales and high contrast. To solve these problems, we often adopt some forms of model reduction such as upscaling and multiscale methods. These methods can reduce the degrees of freedom of a problem. Most of the existing techniques are based on the so called offline construction. In particular, reduced models are computed in a pre-processing step, called offline stage, before the actual simulations, called online stage, are performed. For example, some multiscale basis functions are pre-computed for multiscale finite element methods. While these methods are effective in a wide variety of applications, there are still situations for which these offline basis are inadequate to give reliable solutions unless a large dimensional offline space is employed. Some of these situations involve external source effects and distant effects, which are ignored by most multiscale model reduction methods since they are typically based on local constructions. Therefore, it is evident that some online computations are necessary. Our new method is based on a combination of offline technique and an online enrichment technique. The online technique is able to produce a reduced model taking care of external sources and distant effects, without using global models. The online construction is also performed locally and adaptively in regions with more heterogeneities, giving very efficient reduced models. We also prove the convergence of our method, which is independent of the contrasts. (This work is partially supported by Hong Kong RGC General Research Fund Project 400411.)

Optimal Symmetry Results for the Optimizers of Caffarelli-Kohn-Nirenberg Inequalities

Maria J. Esteban (University Paris-Dauphine)

In this talk I will present recent results, obtained in collaboration with J. Dolbeault and M. Loss, proving the radial symmetry of the optimizers of Caffarelli-Kohn-Nirenberg inequalities whenever they are local minima in the full functional space. These results are optimal and close a series of works proving partial results. The method used to obtain this result, which actually proves the uniqueness of positive solutions for the corresponding Euler-Lagrange equations, is based on a nonlinear fast diffusion flow which is applied around any positive solution to explore the energy landscape around it.

The Stability of Laminar Shear Flow

Weinan E (Princeton University, Peking University)

In 1883, Reynolds published his classical work on the experimental study of the stability of shear show and transition to turbulence. Since then the issue of the critical Reynolds number at which laminar flows become unstable has been studied by numerous people, includig Sommerfeld, Heisenberg, C. C. Lin, Orazag, and more recently, Trefethen, Hof, Barkley, Eckhardt, Walefe et. al. Despite this great deal of effort, there is still a lack of the proper mathematical framework for understanding such nonlinear instabilities. In this talk, we present a theoretical framework using ideas drawn from statistical physics and large deviation theory. We then present the results of our intensive numerical study of the shear flow using this framework.

Self-Similar Singularity for a 1D Model of the 3D Axisymmetric Euler Equations

Tom Hou (California Institute of Technology)

We investigate the self-similar singularity for a 1D model of the 3D axisymmetric Euler equations, which is motivated by a particular singularity formation scenario observed in a recent numerical computation. Using a very delicate method of analysis which involves computer aided proof, we prove the existence of a discrete family of self-similar profiles for this model and analyze their far-field properties. The self-similar profiles we obtain agree with those obtained by direct numerical simulations of the model. Moreover, the self-similar profile seems to enjoy some stability property. The self-similar profiles we construct are non-conventional in the sense that they do not decay to zero at infinity but grow with certain fractional power. Such behavior is also observed in the recent numerical computation of the 3D Euler equations of Luo and Hou, which is very different from the Leray type of self-similar solutions of the 3D Euler equations.

Modelling and Simulation in 3-d of Compressible Plasma Flow in a High Current Circuit Breaker

Rolf Jeltsch (ETH, Switzerland)

The main function of a circuit breaker is to switch off the electric current safely, in case of fault current. A mechanical force separates the contacts, and an arc starts to burn between the two contacts. This plasma is described by the resistive Magneto hydro dynamics (MHD) equations. The emphasis is on very high currents (10kA-200kA) and relatively high conductivity. Radiation is incorporated by adding a Stefan's radiation. To simulate the plasma in the arc the Nektar code developed by Brown University is adapted and extended. It is based on the Discontinuous Galerkin(DG) methods allowing for hexagonal or tetrahedral meshes in 3d. GID is used for mesh generation. The code is extended to include Runge-Kutta time stepping, various accurate Riemann solvers for MHD, slope limiters and SF 6 gas data. It operates on both serial and parallel computers with arbitrary number of processors. The suitability of this Runge-Kutta Discontinuous Galerkin (RKDG) methods is analysed. In particular different numerical fluxes, different Riemann solvers and limiters, low and high order approximations on smooth and non-smooth solutions are investigated. Numerical results are given. This work is the result of a larger team. However the 3d simulations have been performed by Harish Kumar in his Ph.D. thesis.

The impact of L1 optimization in Nonlinear PDE

Stan Osher (UCLA), joint with many people including J. Darbon, V. Ozolins, R. Caflisch, H. Schaeffer, W. Feldman, C. Hauck and G. Tian

At this time almost everyone interested in finding sparse solutions to discrete equations is aware that l1 optimization plays a key role. However that fact that L1 optimization, using the techniques developed for l1, is a very powerful tool in nonlinear PDE and numerical analysis is less widely known. H. Brezis, in 1974, showed that adding an L1 type penalty to calculus of variations problems which lead to elliptic equations plus a signum type terms gives solutions with compact support. I will discuss this, numerical aspects, applications to Schrodinger equations, obstacle problems, numerical homogenization and certain high dimensional PDE's.