第二届偏微分方程数值方法研究生论坛暨首届北京计算数学研究生论坛及偏微分方程数值方法研讨会

主办单位

北京国际数学研究中心

北京计算数学学会

资助单位

北京大学研究生院

北京市科学技术协会

北京国际数学研究中心

学术委员会

许进超（主席）	北京大学, Penn State
胡　俊	北京大学
黄忠亿	清华大学
李　若	北京大学
王双虎	北京应用物理与计算数学研究所
张林波	中国科学院数学与系统科学研究院

组织委员会

胡 俊（主席）	北京大学
樊玉伟	北京大学
胡凯博	北京大学
李小平	北京大学
马 睿	北京大学
吴开亮	北京大学

联系人

李小平

Email：lxping.ynufe@gmail.com

电 话：18811788763

会议地址：北京大学理科一号楼1114M

研究生论坛日程（2014-7-13）

8:00-12:00	主持人	胡俊
8:00-8:20	许进超	论坛致词
8:20-8:40	李　想	油藏模拟问题的求解及线性求解器的设计
8:40-9:00	陈艳利	非匹配网格上二阶椭圆方程自适应有限体积法
9:00-9:20	韩家宇	A new adaptive mixed finite element method based on residual type a posterior error estimates for the Stokes eigenvalue problem
9:20-9:40	姜勇越	Numerical Simulation for Multiphase Power-Law Fluids with Moving Contact Line Using Continuous Finite Element Schemes
9:40-10:00	高　斯	修正虚拟介质方法在流体与弹塑性固体耦合中的应用
10:00-10:20		茶歇
10:20-10:40	李志文	High Order Galerkin Methods with Graded Meshes for Two-dimensional Reaction-Diffusion Problems
10:40-11:00	胡凯博	Structure-Preserving and Energy Stable Discretization of MHD Equations
11:00-11:20	杜　涛	A fast algorithm for blood flow in arterial trees
11:20-11:40	马　睿	Guaranteed Lower and upper bounds for eigenvalues of second order elliptic operators in any dimension
11:40-12:00	高艳妮	三温辐射能量方程组的自适应计算
12:00-14:00		午餐
14:00-15:40	主持人	马睿
14:00-14:20	吴开亮	Third-Order Accurate Direct Eulerian GRP Schemes for Compressible Flows
14:20-14:40	刘利斌	奇异摄动问题的数值方法研究及网格参数估计
14:40-15:00	易雯帆	An Augmented Singular Transform and its Partial Newton Method for Finding New Solutions
15:00-15:20	张国栋	Decoupled schemes for MHD equations
15:20-15:40	许小静	Hybrid stress quadrilateral finite element approximation for stochastic plane elasticity equations

15:40-16:00		茶歇
16:00-17:20	主持人	吴开亮
16:00-16:20	李　朗	基于 BCS-BEC 交叉区域的 Ginzburg-Landau 方程的 Fourier 谱方法
16:20-16:40	周歆辰	A new 8-DoF quadratic nonconforming quadrilateral finite element
16:40-17:00	孙　琪	基于 CVT 方法建立 Navier-Stokes 方程降阶模型
17:00-17:20	谷淑婷	An energy-stable finite-difference scheme for the binary fluid-surfactant system

偏微分方程数值方法研讨会

日程（2014-7-14）

8:30-11:30	Jinchao Xu	Chair
8:30-9:00	Wenjun Ying	Recent Developments of the Kernel-free Boundary Integral Method for PDEs
9:00-9:30	Junming Xiao	An iterative method for computing Beltrami fields
9:30-10:00	Buyang Li	Error estimates of fully discrete FEMs for nonlinear parabolic equations without grid-ratio conditions
10:00-10:30	**Tea break**	
10:30-11:00	Shipeng Mao	Adaptive finite element method for incompressible magnetohydrodynamics
11:00-11:30	Xuehai Huang	A superconvergent C^0 discontinuous Galerkin method for Kirchhoff plates: error estimates hybridization and postprocessing
11:30-13:30	**Lunch**	
13:30-16:00	Jun Hu	Chair
13:30-14:00	Hehu Xie	Some Multilevel Correction Methods for Eigenvalue Problems
14:00-14:30	Hongling Hu	Extrapolation cascadic multigrid method on piecewise uniform grid
14:30-15:00	**Tea break**	
15:00-15:30	Yuanming Xiao	An Immersed Finite Element Method for Problems Involving Curved Interface or Boundary
15:30-16:00	Yifeng Xu	Convergence of an Adaptive Finite Element Method for Distributed Flux Reconstruction

Titles and Abstracts

油藏模拟问题的求解及线性求解器的设计

李想,北京大学

经过近四十年的发展，油藏模拟已经成为一项成熟的技术，在现代石油工业中，油藏模拟处于数据处理的核心地位，是进行生产历史拟合、产量预测以及生产制度优化必不可少的工具。油藏模拟是用数值方法求解油藏物质平衡、化学平衡与热平衡的偏微分方程，经典的求解方案是用有限差分离散并用牛顿法求解非线性方程组。如今，随着油藏模型规模的不断扩大，以及所模拟的物理和化学过程的越来越复杂，模型的网格数量及非线性程度也在不断增长，这对求解器的效率提出了新的要求。

油藏方程的 Jacobian 矩阵具有椭圆（压力变量）和双曲（饱和度变量）的双重性质，工业上常用基于 CPR（Constrained Pressure Residual）的多级预条件方法迭代求解。本研究在北京工学院自主开发的油藏模拟器 UNCONG 上实现了基于 CPR 的迭代求解器，对比了不同的压力求解方法、不同的解耦方式对求解速度的影响，并针对与油藏耦合的复杂井，设计了特殊的解耦、求解方案。本方案的速度领先于许多其他求解器，在 SPE10 两相模型中，一个牛顿步的耗时约为 4 秒，在 SPE10 黑油模型中，一个牛顿步的耗时约为 7 秒。为充分发挥硬件效能，我们还对求解器乃至整个模拟器实现了基于共享内存的并行，研究发现，自由度低的模型（如黑油模型）并行效率较低，而自由度高的模型（如组分模型）并行效率较高，对组分模型 SPE3，四线程并行可达到 2.7 倍的速度。

本研究提出的求解方案效率达到了业内领先，且具有良好的通用性（无网格、不依赖变量选取、适用于有复杂井的问题）和一定的可扩展性。

非匹配网格上二阶椭圆方程自适应有限体积法

陈艳利,吉林大学

在本文中我们构造和分析了两种非匹配网格上二阶椭圆方程自适应双线性有限体积法.其中一种选择在网格匹配处连续,网格不匹配处间断的分片双线性有限元空间作为试探函数空间;另一种选择间断分片双线性有限元空间作为试探函数空间.我们给出了先验和后验误差估计.最后,给出数值例子验证方法的有效性。

A new adaptive mixed finite element method based on residual type a posterior error estimates for the Stokes eigenvalue problem

韩家宇, 贵州师范大学

We combine mixed finite element method, multiscale discretization and Rayleigh quotient iteration to propose a new adaptive algorithm based on residual type a-posterior error estimates for the Stokes eigenvalue problem. Both reliability and efficiency of the error indicator are proved. The efficiency of the algorithm is also investigated by numerical experiments using iFEM package.Numerical results are satisfying.

Numerical Simulation for Multiphase Power-Law Fluids with Moving Contact Line Using Continuous Finite Element Schemes

姜勇越, 北京科技大学

In this paper, we investigate the motion of isothermal multiphase incompressible power-law fluids. The governing equations are Navier-Stokes-Cahn-Hilliard coupled models with the generalized Navier boundary condition proposed by Qian, Wang and Sheng in 2003. The continuous finite element method is used to discretize the governing equations in space and a modified midpoint scheme in time. In order to increase the stability in the pressure variable, a penalty item is added in the continuity equation. We derive the discrete energy law having the similar type as the continuous energy law. Two kinds of immiscible fluids in a pipe with a moving contact line is studied as the numerical example and also this example is computed with a relatively coarse grid under this continuous finite element scheme preserving energy law. The accuracy of the scheme are validated by example and result.

修正虚拟介质方法在流体与弹塑性固体耦合中的应用

高斯, 北京航空航天大学

采用修正虚拟介质方法（MGFM）处理流体与弹塑性固体耦合问题， 其中流体为可压缩气体或液体，固体为带有 von Mises 屈服条件的理想弹塑性固体。在流固耦合界面处定义一维流固多介质黎曼问题，并通过相应的黎曼问题求解器确定虚拟介质状态，即可实现用修正虚拟介质方法对该流固耦合问题解耦。在一维和二维的应用中，给出了水与铝相互作用的数值结果。修正虚拟介质方法表现出简单、可靠等优点。

High Order Galerkin Methods with Graded Meshes for Two-dimensional Reaction-Diffusion Problems

李志文, 中山大学

We develop high-order Galerkin methods with graded meshes for solving the two-dimensional reaction-diffusion problem on a rectangle. With the help of the comparison principle, we establish upper bounds for high order partial derivatives of an arbitrary order of its exact solution. According to prior information of the high order partial derivatives of the solution, we design both implicit and explicit graded meshes which lead to numerical solutions of the problem having an optimal convergence order. Numerical experiments are presented to confirm the theoretical estimate and to demonstrate the outperformance of the proposed meshes over the Shishkin mesh.

Structure-Preserving and Energy Stable Discretization of MHD Equations

胡凯博, 北京大学

In this presentation, we report some structure-preserving and energy-stable finite element methods for solving the incompressible MHD systems. One goal is to preserve some key divergence-free conditions strongly on the discretized level by means of appropriate mixed formulations and appropriate finite element spaces for various physical variables. Furthermore, we establish the energy estimate on the discretized level and show that the proposed numerical scheme is an energy-stable discretization for the incompressible MHD system. Uniform wellposedness with respect to small time step size will be shown based on inf-sup condition.

A fast algorithm for blood flow in arterial trees

杜涛, 上海交通大学

Nonlinear one-dimensional models of the interaction between blood flow and elastic vessel walls are widely used in studying the pulse propagation in arterial trees. Classical finite difference schemes, such as Lax-Wendroff scheme, are used to compute the blood pressure and flow rate in the nonlinear one-dimensional model. However, the computational cost is very high due to the large wave speed in artery. In this work, we obtain the two Riemann variables along the two opposite characteristic directions based on the one-dimensional model. Meanwhile, temporal-to-spatial coordinate is replaced by spatial-to-temporal coordinate. This conversion allows a much bigger temporal grid size than the original finite difference method, in respect that reciprocal of wave speed is rather small due to the large wave

speed of blood flow in arteries. With initial value of the Riemann variables from the linearized model of the one-dimensional model, a iterative method is used to compute the Riemann variables in the arterial tree. Based on the compression of error of outlet boundary condition in one vessel, a similar scheme is designed for updating the Riemann variables at the outlet of the parent vessel of each bifurcation. The compression of error at outlet of each vessel ensures the convergence of the iteration. The results from the new fast scheme almost overlap with the results from the classical finite difference method, and computational efficiency of the new method is much higher, over 100 times,than the classical finite difference method.

Guaranteed Lower and upper bounds for eigenvalues of second order elliptic operators in any dimension

马睿, 北京大学

A new method is proposed to produce guaranteed lower bounds for eigenvalues of general second order elliptic operators in any dimension. Unlike most methods in the literature, the proposed method only needs to solve one discrete eigenvalue problem but not involves any base or intermediate eigenvalue problems, and does not need any a priori information concerning exact eigenvalues either.Moreover, it just assumes basic regularity of exact eigenfunctions.This method is defined by a novel generalized Crouzeix-Raviart element which is proved to yield asymptotic lower bounds for eigenvalues of general second order elliptic operators, and a simple post-processing method.As a byproduct, a simple and cheap method is also proposed to obtain guaranteed upper bounds for eigenvalues,which is based on generalized Crouzeix-Raviart element approximate eigenfunctions,an averaging interpolation from the generalized Crouzeix-Raviart element space to the conforming linear element space,and an usual Rayleigh-Ritz procedure.The ingredients for the analysis consist of a crucial projection property of the canonical interpolation operator of the generalized Crouzeix-Raviart element,explicitly computable constants for two interpolation operators.Numerics are provided to demonstrate the theoretical results.

三温辐射能量方程组的自适应计算

高艳妮, 吉林大学

耦合光子、电子、离子三种温度的辐射能量方程组，具有强间断、强耦合、强非线性的特点，这给该方程组的高效数值模拟增加了难度。在三角网线性元有限体积法的背景下，结合相应的残量型后验误差估计，我们构造了两种适用于该问题的空间网格自适应算法。在实际的数值模拟中，我们发现这两种自适应算法均能大量减少计算体积和计算时间，并且能保证较高的数值精度。

Third-Order Accurate Direct Eulerian GRP Schemes for Compressible Flows

吴开亮, 北京大学

The generalized Riemann problem (GRP) scheme, as an analytic extension of the Godunov method, is originally devised by utilizing a piecewise linear function to approximate the initial data and then analytically solving a local GRP at each interface so as to yield numerical fluxes.

In this talk, we will introduce the third-order accurate direct Eulerian GRP schemes for 1D and 2D Euler equations as well as the 1D special relativistic hydrodynamical equations. In the 1D GRP schemes, the higher-order WENO initial reconstruction is employed, and the local GRPs in the Eulerian formulation are directly and analytically resolved to third-order accuracy via the Riemann invariants and Rankine-Hugoniot jump conditions, to get the approximate states in numerical fluxes. The 2D GRP scheme is implemented by using the third-order accurate time-splitting method. Several numerical results will be shown to demonstrate the accuracy and the performance of the proposed GRP schemes.

This is a joint work with Dr. Zhicheng Yang and Prof. Huazhong Tang.

奇异摄动问题的数值方法研究及网格参数估计

刘利斌, 华南师范大学

首先，对奇异摄动问题的层适应网格方法进行了研究，讨论了其网格参数估计的问题，并给出了最优参数估计的方法. 然后，研究了奇异摄动微分方程组的自适应移动网格方法, 给出了网格控制函数和网格生产算法，并给出了一致收敛性分析.

An Augmented Singular Transform and its Partial Newton Method for Finding New Solutions

易雯帆, 湖南师范大学

Using the information provided by previously found solutions, a new augmented singular transform is developed to change the local basin structure in the original problem for finding new solutions. Then an augmented partial Newton method is designed to solve the augmented problem on the solution set. Mathematical justification of the new formulation and method is established. Details of algorithm implementation are presented. Numerical results for two very different variational problems are obtained and displayed with their profile-contour plots and data on convergence and errors.

Decoupled schemes for MHD equations

张国栋, 西安交通大学

We devise decoupled numerical methods for MHD equations, discuss their stability and convergence, and several numerical examples are presented.

Hybrid stress quadrilateral finite element approximation for stochastic plane elasticity equations

许小静, 四川大学

We consider finite element analysis of plane elasticity equations with stochastic Young's modulus and stochastic loads. Firstly, we apply Karhunen-Loève expansion to stochastic Young's modulus and stochastic loads so as to turn the original problem into a system containing a finite number of deterministic parameters.Then we deal with the stochastic domain by k-version or p-version finite element approximation and the space domain by a hybrid stress quadrilateral finite element method. Uniform a priori error estimates are derived in the sense that the error bound is independent of the relevant Lamé constant λ. Finally, we verify the theoretical results by some numerical experiments.

基于 BCS-BEC 交叉区域的 Ginzburg-Landau 方程的 Fourier 谱方法

李朗, 华南农业大学

本文将研究如下在 BCS-BEC 交叉区域关于原子费米气体的 Ginzburg-Landau 方程

$$\begin{cases} idu_t = \left(-\dfrac{dg^2+1}{U}+a\right)u + g\left[a+d\left(2v-2\mu\right)\right]\varphi + \dfrac{c}{4m}\Delta u \\ \qquad\qquad + \dfrac{g}{4m}\left(c-d\right)\Delta\varphi - b\left|u+g\varphi\right|^2\left(u+g\varphi\right), \\ i\varphi_t = -\dfrac{g}{U}u + \left(2v-2\mu\right)\varphi - \dfrac{1}{4m}\Delta\varphi, \\ u\left(x,0\right)=u_0\left(x\right), \varphi\left(x,0\right)=\varphi_0\left(x\right), \\ u\left(x+2\pi,t\right)=u\left(x,t\right), \varphi\left(x+2\pi,t\right)=\varphi\left(x,t\right). \end{cases}$$

本文主要采用 Fourier 谱方法对其进行数值研究，结合离散形式下的插值不等式、Gronwall 不等式等工具证明了数值解的收敛性，并给出数值实验验证了数值结果与我们的理论是一致的。

A new 8-DoF quadratic nonconforming quadrilateral finite element

周歆辰, 大连理工大学

We propose a new quadratic quadrilateral nonconforming finite element consisting of 8 degrees of freedom (DoFs). The construction procedure is to enforce a linear constraint involving the first order moments on the edges of a quadrilateral based on a nonparametric 9-DoF element. We mainly consider the approximations of 2-dimensional second order elliptic problems and stationary Stokes equations using our new element over arbitrary convex quadrilateral meshes. For elliptic problems, error estimates with optimal convergence order in both broken H^1 norm and L^2 norm are given. For Stokes equations, we adopt our new element to approximate each component of the velocity, along with piecewise discontinuous P_1 element for the pressure. This mixed scheme is shown to be stable and optimal error estimates both for the velocity and the pressure are also achieved. Numerical examples verify our theoretical analysis.

基于 CVT 的 Navier-Stokes 方程降阶模型

孙琪, 北京计算科学研究中心

利用 CVT(Centroidal Voronoi Tessellation)方法建立了 Navier-Stokes 方程的降阶模型。

An energy-stable finite-difference scheme for the binary fluid-surfactant system

谷淑婷, 北京师范大学

We present an unconditionally energy stable finite-difference scheme for the binary fluid-surfactant system. The proposed method is based on the convex splitting of the energy functional with two variables. Here are two distinct features: (i) the convex splitting energy method is applied to energy functional with two variables, and (ii) the stability issue is related to the decay of the corresponding energy. The full discrete scheme leads to a decoupled system including a linear sub-system and a nonlinear sub-system. Algebraic multigrid and Newton-multigrid methods are adopted to solve the linear and nonlinear systems, respectively. Numerical experiments are shown to verify the stability of such a scheme.

Recent Developments of the Kernel-free Boundary Integral Method for PDEs

Wenjun Ying,Shanghai Jiao Tong University

The kernel-free boundary integral (KFBI) method is a Cartesian grid-based method for solving elliptic PDEs on complex domains. It solves elliptic PDEs in the framework of boundary integral equations. Provided that the elliptic PDE is reformulated as a Fredholm boundary integral equation/system of the second kind, which is well-conditioned, the discrete equations can be efficiently solved by a Krylov subspace iterative method with the iteration number essentially independent of the mesh parameter and system dimension. In general, the boundary integrals in the linear system can NOT be evaluated by the standard method since the integral kernels and the Green's function of the elliptic PDE may be difficult to compute or at least not directly available, for example, when the PDE has variable coefficients. In the Krylov iteration for the boundary integral equation/system, evaluation of the boundary or volume integrals is done with the kernel-free boundary integral method. To evaluate a boundary or volume integral encountered, the KFBI method first solves an equivalent interface problem on a rectangle, which embeds the complex domain or interface and is partitioned into a Cartesian grid, with a fast Fourier transform (FFT) or geometric multigrid based fast solver. It then interpolates the discrete solution on the Cartesian grid to get values of the boundary or volume integral at discretization points of the domain boundary or interface. In this way, the boundary/volume integrals are approximately and indirectly evaluated while there is no need to know any analytical expression of the integral kernel or the Green's function of the elliptic PDE. The KFBI method takes full advantages of 1) the well-conditioning of the reformulated boundary integral equation/system, 2) the efficiency of fast elliptic solvers for the equivalent interface problems on Cartesian grids, and 3) avoids generation of any unstructured body-fitted volume or surface grids. In this talk, recent developments of the kernel-free boundary integral method for elliptic boundary value problems, interface problems, time-dependent reaction-diffusion problems as well as moving interface and free boundary problems will be presented.

An iterative method for computing Beltrami fields

Junming Xiao,Chinese Academy of Science

In this paper, we are concerned with numerical methods for solving the Beltrami field equations. By introducing a new variable, we transform the original nonlinear problem into a minimization problem involving two unknowns. We consider the edge finite element discretization of the minimization problem. We first prove the existence of solutions of the discrete minimization problem, and present the decay of the objective functional at the global minimal point with the mesh size decreasing. Then we design an iterative method for solving the discrete minimization problem so that the two unknowns can be solved

respectively in each iterative step. The solvability of the problem in each step is proved. Moreover, the finite termination of the numerical method is shown. As we will see, the Fréchet derivative of the objective functional is vanishing with the number of iterative steps increasing. The numerical results indicate that the iterative method proposed is effective and reliable.

Error estimates of fully discrete FEMs for nonlinear parabolic equations without grid-ratio conditions

Buyang Li,Nanjing University

We survey some previous works on error estimates of fully discrete finite element methods for some nonlinear parabolic equations from physics and engineering. Due to the nonlinear structure, the traditional approach often requires certain restrictions on the grid ratio (between the time-step size and spatial mesh size) in establishing error estimates, even for implicit or semi-implicit time-stepping schemes. We introduce a new approach to avoid these restrictions on the grid ratio. Then we discuss the possibility of extending this new approach to other time-stepping schemes and other spatial discretizations, as well as nonlinear parabolic equations with nonsmooth structure. Several open questions related to this topic will also be discussed.

Adaptive finite element method for incompressible magnetohydrodynamics

Mao Shipeng,Chinese Academy of Science

We consider a mixed finite element method for the numerical discretization of a stationary incompressible magnetohydrodynamics problem in three dimensions with its velocity field is discretized using H^1 conforming elements and the magnetic field is approximated by curl-conforming Nédélec elements. Under the assumption that the original model has a unique solution pair, we derive a posteriori error estimates of the incompressible magnetohydrodynamic (MHD) equations with a sharp upper bound. Using these a posteriori error estimates, we construct an adaptive algorithm for computing the solution of 3D magnetohydrodynamics. Numerical experiments are carried out to show the performance of the adaptive finite element method.

A superconvergent C^0 discontinuous Galerkin method for Kirchhoff plates: error estimates hybridization and postprocessing

Xuehai Huang,Wenzhou University

A superconvergent C^0 discontinuous Galerkin (SCDG) method for Kirchhoff plates is proposed in this paper, where piecewise polynomial of degree *k-1* and *k-th* Lagrangian element are used to approximate

moment and deflection respectively. With the help of concepts of normal bending moment and twisting moment, we first propose a new framework of CDG methods for Kirchhoff plates, and then acquire the SCDG method by choosing suitable numerical traces carefully. Some optimal and superconvergent error estimates are proved using two interpolation operators and duality argument. Under some assumption on stabilization parameters, we consider the hybridization of SCDG method. Furthermore, a new discrete deflection is constructed by postprocessing the solution of SCDG method,which superconverges to deflection with order $k + 1$ in broken H^1 norm. Finally, some numerical results are shown to demonstrate the theoretical results.

Some Multilevel Correction Methods for Eigenvalue Problems

Hehu Xie, The State Key Laboratory of Scientific and Engineering Computing

In this lecture, we will presents some ways which may leads to the multilevel correction methods. These methods are based on the Aubin-Nitsche of the finite element method or shift inverse power method or Newton iteration method for eigenvalue problems. Some properties of the multilevel correction methods will also be discussed.

Extrapolation cascadic multigrid method on piecewise uniform grid

Hongling Hu,Hunan Normal University

The triangular linear finite elements on piecewise uniform grid for an elliptic problem in convex polygonal domain are discussed. Global superconvergence in discrete H^1-norm and global extrapolation in discrete L^2-norm are proved. Based on these global estimates the conjugate gradient method (CG) is effective, which is applied to extrapolation cascadic multigrid method (EXCMG). The numerical experiments show that EXCMG is of the global higher accuracy for both function and gradient.

An Immersed Finite Element Method for Problems Involving Curved Interface or Boundary

Yuanming Xiao,Nanjing University

Firstly, we propose novel immersed finite element methods for elliptic and Maxwell problems involving curved interface and boundary. The meshes in our methods do not need to fit the interface or boundary. New error indicators are introduced to control the error due to non-body-fitted meshes. Flexible h-adaptive strategies are developed, which could be naturally extended to time-dependent problems. Special attention is then paid on tackling parabolic problems in time varying domains based on the

aforementioned method. Extensive numerical experiments are performed to support the theoretical results and to show the competitive behavior of the adaptive algorithm for problems defined on non-smooth domains. This is a joint work with Zhiming Chen, Linbo Zhang and Zedong Wu.

Convergence of an Adaptive Finite Element Method for Distributed Flux Reconstruction

Yifeng Xu, Shanghai Normal University

In this talk, I talk about convergence of an adaptive finite element method for the reconstruction of the distributed flux in a diffusion system. The adaptive method is based on an posteriori error estimator for the distributed flux, state and costate variables. It is demonstrated that the sequence of discrete solutions produced by the adaptive algorithm with the maximum marking strategy converges to the true triplet satisfying the optimality conditions in the energy norm and the corresponding error estimator converges to zero asymptotically. The results can be extended to other practical marking strategies. One numerical example is given to verify the theoretical result. This is a joint work with Prof. Jun Zou at The Chinese University of Hong Kong.