

MINI-COURSES ON CONFORMAL GEOMETRY AND
GEOMETRIC PDE
JUNE 22 - JUNE 26 2015

1. Lecturers

Matthew Gursky (University of Notre Dame)

Fengbo Hang (New York University)

Mara del Mar Gonzalez Noguera (Universitat Politecnica de Catalunya)

Guofang Wang (University of Freiburg)

Juncheng Wei (University of British Columbia)

2. Titles and Abstracts

Matthew Gursky, University of Notre Dame

Title: Critical metrics of quadratic curvature functionals

Abstract: In these lectures I will give an overview of a proof of existence, via gluing methods, of metrics which are critical points of quadratic Riemannian functionals.

This is a joint project with J. Viaclovsky.

Fengbo Hang, New York University

Title: Q curvature on three manifolds

Abstract: The study of fourth order Paneitz operator and associated Q curvature improves our knowledge of four dimensional conformal geometry. Here we will describe some recent progress in understanding of the Paneitz operator in dimension three. Among other things we will discuss the associated Sobolev inequality, sign of Green's function and existence and compactness of constant Q curvature metrics.

Mara del Mar Gonzalez Noguera, Universitat Politecnica de Catalunya

Title: An introduction to the conformal fractional Laplacian and the fractional Yamabe problem.

Abstract: The conformal fractional Laplacian is a pseudo-differential operator on the conformal infinity of a conformally compact Einstein manifold, and it is constructed through scattering theory. Fractional order curvature is defined from the conformal fractional Laplacian and can be understood a non-local generalization of mean curvature. In this minicourse we will formulate the (fractional) Yamabe problem and give the solution in several cases. Although the operator is nonlocal, the main idea is to write a local extension problem that can be handled through elliptic methods.

Guofang Wang, University of Freiburg

Title: The Gauss-Bonnet-Chern curvature and its related problems

Abstract: In this minicourse I will first introduce the Gauss-Bonnet-Chern curvature and discuss its properties. It is a natural generalization of the scalar curvature and has two interesting special cases: If the manifold is a hypersurface in the Euclidean space, it is the higher order mean curvature; if the manifold is locally conformally flat, it is the so-called δ_k -scalar curvature, which has been intensively studied in the field of fully nonlinear equations. Then I will use it to define a geometric invariant for asymptotically flat and asymptotically hyperbolic manifolds, which is a natural

generalization of the ADM mass. Its corresponding Penrose inequality is closely related to the classical inequalities, the Alexandrov-Fenchel inequalities, which I will talk about at the end of this minicourse.

Juncheng Wei, University of British Columbia

Title: On Serrin's overdetermined problem in unbounded domains and Berestycki-Caffarelli-Nirenberg Conjecture

Abstract: We consider the following overdetermined problem

$$\begin{aligned}\Delta u &= f(u), u > 0 \text{ in } \Omega \\ u &= 0 \text{ on } \partial\Omega \\ \partial_{\nu}u &= C \text{ on } \partial\Omega\end{aligned}$$

Serrin proved (1971) that if the domain is bounded then the domain must be a ball and that u must be radially symmetric. This talk is concerned with the case when Ω is unbounded. In 1997, Berestycki, Caffarelli and Nirenberg conjectured that if Ω^c is connected and Serrin's overdetermined problem admits a solution then Ω is either a half space, or a cylinder, or the complement of a ball. In this talk we present results of BCN conjecture in both positive and negative answers. We first show that if the domain Ω is an epigraph then Ω must be an half space, provided that one of the following assumptions are satisfied: either $n = 2$, or the graph is globally Lipschitz; or $n \leq 8$ and $\frac{\partial u}{\partial x_n} > 0$ in Ω . Then we show that this result is optimal by constructing an epigraph in dimensions $n \geq 9$, which is a perturbation of Bombieri-De Giorgi-Giusti minimal graph, such that Serrin's overdetermined problem admits a solution. Furthermore we establish an intricate relation between Serrin's overdetermined problem and CMC surfaces. We prove that for any nondegenerate CMC surfaces or minimal surfaces Serrin's overdetermined problem is solvable.

Shen Weiming, Peking University

Title: Asymptotic behavior of Lowner-Nirenberg problem in singular domains

Abstract: The Lowner-Nirenberg problem, which is also known as the singular Yamabe problem, is related to the existence of a complete conformal metric with constant scalar curvature on domains in \mathbb{R}^n .

In this talk, we will discuss the asymptotic behavior of solutions of the Lowner-Nirenberg problem in singular domains and prove that the solutions are well approximated by the corresponding solutions in tangent cones defined on singular points on boundary. This is a joint work with Professor Han Qing.

Jiang Xumin, Notre Dame

Title: Boundary expansion for Kahler Einstein metrics in the pseudoconvex domain

Abstract: In this talk, we discuss boundary expansions for complete Kahler Einstein metrics in bounded strictly pseudoconvex domains. We developed an ODE iteration method to discuss the remainder estimates in the context of local finite regularity and the convergence of the expansions under a natural condition that the boundary is analytic.

Ma Shiguang, Nan Kai University

Title: UNIQUENESS THEOREMS FOR FULLY NONLINEAR CONFORMAL EQUATIONS ON SUBDOMAINS OF THE SPHERE.

Abstract: In this paper, the authors prove classification results to elliptic fully nonlinear conformal equations on certain subdomains of the sphere with prescribed constant mean curvature on its boundary.

Sean Curry, University of Auckland

Title: Constructing Invariants of CR Embeddings Using Tractors

Abstract: CR geometry and CR embeddings arise naturally in the study of proper holomorphic mappings between smoothly bounded domains in several complex variables. CR invariants of the boundaries of domains are used to tell whether two domains are biholomorphically equivalent.

The main way of producing such invariants involves using the CR Cartan (or Chern-Moser) connection. We apply this connection (in 'tractor' form) to the problem of constructing invariants of CR embeddings between CR manifolds. This extends ideas that have been useful in producing invariants of conformal embeddings (such as generalized Willmore invariants) to the CR setting.

Ruobing Zhang, Princeton University

Title: Quantitative Nilpotent Structure and ϵ -Regularity on Collapsed Manifolds with Ricci Curvature Bounds

Abstract: In this talk we discuss the ϵ -regularity theorems for Einstein manifolds and more generally manifolds with just bounded Ricci curvature, in the collapsed setting. A key tool in the regularity theory of noncollapsed Einstein manifolds is the following:

If a bigger geodesic ball on an Einstein manifold is sufficiently Gromov-Hausdorff-close to a ball on the Euclidean space of the same dimension, then in fact the curvature on a smaller ball is uniformly bounded. No such results are known in the collapsed setting, and in fact it is easy to see without more such results are false. It turns out that the failure of such an estimate is related to topology. Our main theorem is that for the above setting in the collapsed context, either the curvature is bounded, or the local nilpotent rank drops. There are generalizations of this result to bounded Ricci curvature and even just lower Ricci curvature. This is a joint work with Aaron Naber.

Azahara de la Torre Pedraza, Universidad Politécnic de Cataluña

Title: ODE solutions for the fractional Laplacian equations arising in conformal geometry.

Abstract: We construct some ODE solutions for the fractional Yamabe problem in conformal geometry. The fractional curvature, a generalization of the usual scalar curvature, is defined from the conformal fractional Laplacian, which is a non-local

operator constructed on the conformal infinity of a conformally compact Einstein manifold.

These ODE solutions are a generalization of the usual Delaunay and, in particular, solve the fractional Yamabe problem

$$(-\Delta)^{\gamma} u = c_{n, \gamma} u^{\frac{n+2\gamma}{n-2\gamma}}, u > 0 \text{ in } \mathbb{R}^n \setminus \{0\},$$

with an isolated singularity at the origin.

This is a fractional order ODE for which new tools need to be developed. The key of the proof is the computation of the fractional Laplacian in polar coordinates.

3. Schedule

Monday June 22 2015

Morning: Mini-Courses

8:30 - 9:30 am

Prof. Maria Gonzalez

Title: An introduction to the conformal fractional Laplacian and the fractional Yamabe problem.

9:45 - 10:45 am

Prof. Juncheng Wei

Title: On Serrin's overdetermined problem in unbounded domains and Berestycki-Caffarelli-Nirenberg Conjecture

11:00 - 12:00 pm

Prof. Maria Gonzalez

Title: An introduction to the conformal fractional Laplacian and the fractional Yamabe problem.

Afternoon: Student Seminar

2:00pm-3:00pm

Dr. Shen Weiming

Title: Asymptotic behavior of Lowner-Nirenberg problem in singular domains

3:00pm - 4:00pm

Dr. Jiang Xumin

Title: Boundary expansion for Kahler Einstein metrics in the pseudoconvex domain

Tuesday June 23 2015

Morning: Mini-Courses

8:30 - 9:30 am

Prof. Juncheng Wei

Title: On Serrin's overdetermined problem in unbounded domains and Berestycki-Caffarelli-Nirenberg Conjecture

9:45 - 10:45 am

Prof. Maria Gonzalez

Title: An introduction to the conformal fractional Laplacian and the fractional Yamabe problem.

11:00 - 12:00 pm

Prof. Fengbo Hang

Title: Q curvature on three manifolds

Afternoon: Student Seminar

2:00pm -- 4:00pm

Prof. Ma Shiguang

Title: UNIQUENESS THEOREMS FOR FULLY NONLINEAR CONFORMAL EQUATIONS ON SUBDOMAINS OF THE SPHERE.

Wednesday June 24 2015

Morning: Mini-Courses

8:30 - 9:30 am

Prof. Fengbo Hang

Title: Q curvature on three manifolds

9:45 - 10:45 am

Prof. Matthew Gursky

Title: Critical metrics of quadratic curvature functional

11:00 - 12:00 pm

Prof. Juncheng Wei

Title: On Serrin's overdetermined problem in unbounded domains and Berestycki-Caffarelli-Nirenberg Conjecture

Afternoon: Student Seminar

2:00pm -- 4:00pm

Dr. Sean Curry

Title: Constructing Invariants of CR Embeddings Using Tractors

Thursday June 25 2015

Morning: Mini-Courses

8:30 - 9:30 am

Prof. Matthew Gursky

Title: Critical metrics of quadratic curvature functionals

9:45 - 10:45 am

Prof. Fengbo Hang

Title: Q curvature on three manifolds

11:00 - 12:00 pm
Prof. Matthew Gursky
Title: Critical metrics of quadratic curvature functionals

Afternoon: Student Seminar
2:00pm -- 4:00pm
Dr. Ruobing Zhang
Title: Quantitative Nilpotent Structure and epsilon-Regularity on Collapsed Manifolds with Ricci Curvature Bounds

Friday June 26 2015

Morning: Mini-Courses
8:30 - 9:30 am
Prof. Guofang Wang
Title: The Gauss-Bonnet-Chern curvature and its related problems

9:45 - 10:45 am
Prof. Guofang Wang
Title: The Gauss-Bonnet-Chern curvature and its related problems

11:00 - 12:00 pm
Prof. Guofang Wang
Title: The Gauss-Bonnet-Chern curvature and its related problems

Afternoon: Student Seminar
2:00pm—3:00pm
Dr. Azahara de la Torre Pedraza
Title: ODE solutions for the fractional Laplacian equations arising in conformal geometry

3:00pm -- 4:00pm
Prof. Hu Xue
Title: Bach flat asymptotically locally Euclidean metrics

4. Participants

Junior Researchers

Chuanqiang Chen, Zhejiang University of Technology
Xuezhong Chen, Nanjing University
Yi Fang, University of Science and Technology of China
Andrew Goetz, Duke University
Xue Hu, Jinan University
Dandan Ji, Capital Normal University
Gang Li, Beijing International Center for Mathematical Research

Mijia Lai, Shanghai Jiao-Tong University
Yueh-Ju Lin, University of Michigan
Shiguang Ma, Nankai University
Yuan Wei, UC Santa Cruz

Graduate Students

Sean Curry, University of Auckland
Jianxing Gao, Rutgers University
Yen-Chang Huang, National Central University, Taiwan
Xumin Jiang, University of Notre Dame
Yichao Li, University of Notre Dame
Luxi Liu, Peking University
Yucheng Lu, UC Santa Cruz
Stephen McKeown, University of Washington
Azahara de la Torre Pedraza, Universidad Polit cnica de Catalu na
Xiaojie Shi, Princeton University
Liming Sun, Rutgers University
Tingting Wang, Harbin Institute of Technology
Siyi Zhang, Princeton University
Ruobing Zhang, Princeton University
Jingyang Zhong, UC Santa Cruz
Bo Zhu, Peking University