Sino-French Research Institute in Arithmetic Geometry

Beijing International Center for Mathematical Research
Lecture Hall, Jia Yi Bing Building, 82# Jing Chun Yuan

Schedule and abstracts

Monday June 22

09:00—10:30 Reception

10:30—12:00 **Hugues Randriam**, Hermitian lattices and the analogy between number fields and function fields I

12:00—14:00 Lunch

14:00—15:30 **Antoine Ducros**, Real-valued differential forms and currents on p-adic analytic spaces I

15:30—16:00 Tea break

16:00—17:30 **Frank Loray**, Codimension one foliations on complex varieties I

Tuesday June 23

09:00—10:30 **Antoine Ducros**, Real-valued differential forms and currents on p-adic analytic spaces II

10:30—11:00 Tea break

11:00—12:00 **Fabien Pazuki**, Bad reduction of curves with CM jacobians

12:00—14:00 Lunch

14:00—15:30 **Frank Loray**, Codimension one foliations on complex varieties II

15:30—16:00 Tea break

16:00—17:00 **Enrica Floris**, Invariance of plurigenera for foliations on surfaces

Wednesday June 24

09:00—10:30 **Hugues Randriam**, Hermitian lattices and the analogy between number fields and function fields II

10:30—11:00 Tea break (group photo)

11:00—12:00 **Guillaume Maurin**, TBA

12:00—14:00 Lunch

14:00—15:30 **Antoine Ducros**, Real-valued differential forms and currents on p-adic analytic spaces III

15:30—16:00 Tea break

16:00—17:00 **Shun Tang**, Localization sequence in arithmetic K-theory and the Lefschetz-Riemann-Roch theorem

19:00 Conference dinner

Thursday June 25

09:00—10:30 **Hugues Randriam**, Hermitian lattices and the analogy between number fields and function fields III

10:30—11:00 Tea break

11:00—12:00 **Qing Liu**, Néron models of algebraic curves

12:00—14:00 Lunch

14:00—15:30 **Martín Sombra**, Arithmetic geometry of toric varieties I

15:30—16:00 Tea break

16:00—17:00 **Ziyang Gao**, Recent developments on the André-Oort conjecture

Friday June 26

09:00—10:30 **Patrice Philippon**, Arithmetic geometry of toric varieties II

10:30—11:00 Tea break

11:00—12:00 **Chenyang Xu,** Proof of Veys' conjecture

12:00—14:00 Lunch

14:00—15:30 **José Burgos**, Arithmetic geometry of toric varieties III

15:30—16:30 Tea break

16:00—17:00 **Jean-Pierre Demailly**, On the Green-Griffiths-Lang and Kobayashi conjectures for the hyperbolicity of algebraic varieties

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Schedule and abstracts

Monday June 29

09:00—10:30 **Henri Gillet**, Singular Arithmetic Riemann Roch Ⅰ

10:30—11:00 Tea break

11:00—12:00 **Ruochuan Liu**, Introduction to relative (phi, Gamma)-modules

12:00—14:00 Lunch

14:00—15:30 **Klaus Künnemann**, A tropical approach to non-archimedean Arakelov theory Ⅰ

15:30—16:00 Tea break

16:00—17:30 **Mathilde Herblot**, Diophantine approximation and slopes method Ⅰ

Tuesday June 30

09:00—10:30 **Klaus Künnemann**, A tropical approach to non-archimedean Arakelov theory Ⅱ

10:30—11:00 Tea break

11:00—12:00 **Fei Xu**, Strong approximation with Brauer-Manin obstruction for varieties containing a connected linear algebraic group with compatible action.

12:00—14:00 Lunch

14:00—15:30 **Henri Gillet**, Singular Arithmetic Riemann Roch Ⅱ

15:30—16:00 Tea break

16:00—17:00 **Emmanuel Peyre**, The upgraded version of Batyrev-Manin program

Wednesday July 1

09:00—10:30 **Mathilde Herblot**, Diophantine approximation and slopes method Ⅱ

10:30—11:00 Tea break (group photo)

11:00—12:00 **Maria Carrizosa**, Polarizations on abelian varieties

12:00—14:00 Lunch

14:00—15:30 **Henri Gillet**, Singular Arithmetic Riemann Roch Ⅲ

15:30—16:00 Tea break

16:00—17:00 **Dasheng Wei**, Strong approximation for a normic variety

19:00 Conference dinner

Thursday July 2

09:00—10:30 **Klaus Künnemann**, A tropical approach to non-archimedean Arakelov theory Ⅲ

10:30—11:00 Tea break

11:00—12:00 **Tong Zhang**, Linear series on certain arithmetic 3-folds

12:00—14:00 Lunch

14:00—15:00 **Mathilde Herblot**, Diophantine approximation and slopes method Ⅲ

15:00—15:30 Tea break

15:30—16:30 **Xinyi Yuan**, On the Averaged Colmez Conjecture

José Burgos (Instituto de Ciencias Matemàticas), **Patrice Philippon** (Institut de Mathématiques de Jussieu - Paris Rive Gauche), **Martín Sombra** (Universitat de Barcelona), Arithmetic of toric varieties. Part 1: Algebraic geometry of toric varieties. Part 2: Arakelov invariants of toric varieties. Part 3: Equidistribution of Galois orbits of small points in toric varieties

The geometric properties of toric varieties are described in terms of combinatorial objects such as fans and polytopes having the same dimension, say n, as the toric variety. We present an extension of this dictionary by linking the arithmetic geometry of toric varieties defined over a number field to convex analysis. Here, the arithmetic ingredients are given by (semipositive) metrics on the toric line bundle associated to a toric divisor. Each of these metrics correspond to a continuous concave function on the associated polytope, that we call the local roof function. These functions combine in a global roof function over the polytope and the corresponding height is expressed, similarly to the degree, as $(n+1)!$ times the integral over the polytope of the global roof function. Other arithmetic invariants, such as the essential and algebraic successive minima, and positivity properties are translated in convex analytic terms as well. As an application, we obtain criteria for the equidistribution of the Galois orbits of points of small height in toric varieties.

Maria Carrizosa (Université Claude Bernard – Lyon I), Polarizations on abelian varieties

Let A an abelian variety over a number field K. We know that the number of isomorphism classes of polarizations of given degree of A is finite. We'll discuss the problem of finding an asymptotic bound for this number.

Jean-Pierre Demailly (Université Joseph Fourier), On the Green-Griffiths-Lang and Kobayashi conjectures for the hyperbolicity of algebraic varieties

The Green-Griffiths-Lang conjecture stipulates that for every projective variety X of general type over ${\mathbb C}$, there exists a proper algebraic subvariety of X containing all non constant entire curves $f: {\mathbb C}\to X$. Using the formalism of directed varieties, we prove that this assertion holds true in case X satisfies a strong general type condition that is related to a certain jet-semistability property of the tangent bundle T_X. We then explain how this result can be used to investigate the long-standing conjecture of Kobayashi (1970), according to which a very general algebraic hypersurface of dimension n and degree at least $2n+1$ in the complex projective space ${\mathbb P}^{n+1}$ is hyperbolic.

Antoine Ducros (Université Pierre et Marie Curie), Real-valued differential forms and currents on *p*-adic analytic spaces

This mini-course will be devoted to a presentation of a joint work with Antoine Chambert-Loir, in which we develop kind of a 'harmonic analysis' formalism on Berkovich spaces.
Let me first explain what our work consists of. We define:
- real differential forms of bidegree (p,q) on a Berkovich space X of dimension n;
- the integral of a (n,n) form (with compact support) on X;
- the boundary integral of a (n,n-1) form.
We have Stokes and Green formulas in this context. We define currents by duality, and the Poincaré-Lelong formula holds in this setting. We are also able to associate to a metrized line bundle (L,||.||) a curvature form $c_1(L,||.||)$ (if ||.|| is not smooth, this is not a form in general, but a current). If (L,||.||) comes from a formal model, the

product current $c\_1(L,\|.\|)^n$ is shown to exist and to be a measure, which coincides with a measure previously defined by Chambert-Loir in terms of intersection theory on the special fiber (in his work on *p*-adic equidistribution of points of small height).

I will begin with a presentation of Berkovich's theory. It will be followed by a general description of the main results by Chambert-Loir and myself, and then I will come to the most technical part, giving the precise definitions of differential forms and of our integrals, and sketching some proofs (Poincaré-Lelong formula, computation of $C\_1(L,\|.\|)^n$ when $(L,\|.\|)$ comes from a formal model...).

Enrica Floris (Imperial College of London) Invariance of plurigenera for foliations on surfaces

Recently, Brunella and McQuillan proved some of the main results in birational geometry in the setting of holomorphic foliations on surfaces. In this talk, after giving some preliminary results and definitions, we will discuss to what extent the theorem of Invariance of Plurigenera can be generalized to foliations on surfaces.

Ziyang Gao (Université Paris Sud), Recent developments on the André-Oort conjecture

The André-Oort conjecture is a diophantine problem concerning special subvarieties of mixed Shimura varieties. It predicts that any subvariety of a mixed Shimura variety containing a Zariski dense subset of special points is again a moduli space of some mixed Hodge structures with some Hodge tensors. An interesting example is when the ambient mixed Shimura variety is the universal abelian variety, in which case special points are precisely the points corresponding to torsion points on CM abelian varieties. After the breakthrough of Pila for this conjecture by introducing the o-minimal theory to its study, this conjecture for pure Shimura varieties is reduced to a lower bound for Galois orbits of special points by a series of work of Klingler-Ullmo-Yafave, Pila-Tsimerman and Daw-Orr. As for general mixed Shimura varieties, I reduced the conjecture to the same lower bound. In the proof, a transcendental and a distribution theorem (Ax-Lindemann and its corollary) of independent interest were proved. In my talk I will explain this conjecture, sketch its proof and explain the very recent result of Tsimerman about the lower bound using the Colmez conjecture.

Henri Gillet (University of Illinois at Chicago) Singular Arithmetic Riemann Roch (joint work with C. Soulé of IHES & CNRS)

The "traditional" arithmetic Riemann Roch theorem applies only to proper morphisms between non-singular arithmetic varieties. However given the lack of resolution of singularities for arithmetic varieties, if one has a smooth projective variety over a number field, there is no guarantee that it has a non-singular model over the ring of integers. Thus it is natural to ask how one may extend the arithmetic Riemann Roch theorem to the category of arithmetic varieties that while having non-singular generic fibers, may be singular over finite places. I shall describe a proof of such a "singular" arithmetic Riemann Roch theorem. Our approach is based on a new proof of the "classical" Baum-Fulton-MacPherson singular Riemann-Roch, which uses de Jong's theorem on the existence of non-singular alterations of schemes.

Mathilde Herblot (Université Paris Diderot – Paris VII), Diophantine approximation and slopes method

Klaus Künnemann (Universität Regensburg), A tropical approach to non-archimedean Arakelov theory

We report on joint work with Walter Gubler (Regensburg). We discuss an extension of the work of Chambert-Loir and Ducros who have recently introduced real valued differential forms and currents on Berkovich spaces (see the lectures of Antoine Ducros at this summer school). In analogy to the theory of forms with logarithmic singularities, we enlarge the space of differential forms by so called delta-forms on the non-archimedean analytification of an algebraic variety. This extension is based on an intersection theory for tropical cycles. We describe a generalization of the Poincaré-Lelong formula which allows us to represent the first Chern current of a formally metrized line bundle by a delta-form. We introduce the associated Monge-Ampère measure as a wedge-power of this first Chern delta-form. This measure equals the corresponding Chambert-Loir measure. The $*$-product of Green currents is a crucial ingredient in the construction of the arithmetic intersection product. Using the formalism of delta-forms, we obtain a non-archimedean analogue at least in the case of divisors. We discuss how our formalism can be used to compute non-archimedean local heights of proper varieties.

Qing Liu (Université de Bordeaux), Néron models of algebraic curves (joint work with Jilong Tong)

Let R be a Dedekind domain with K its fraction field. Let X be a separated smooth algebraic variety over K. The Néron model of X is a smooth R-model of X satisfying the so-called universal Néron mapping property. It is well known that when X is an abelian variety, the Néron model of X always exists. In this talk, we will consider the situation when X is a curve. We shall show that in most cases, X admits a Néron model over R.

Ruochuan Liu (Beijing International Center for Mathematical Research),

The theory of (phi, Gamma)-modules is a powerful tool in classical p-adic Hodge theory. In this talk, I will give a survey on my joint work Kedlaya which develops the theory of relative (phi, Gamma)-modules over non-archimedean analytic spaces.

Frank Loray (Université de Rennes I), Codimension one foliations on complex varieties

We give the definition and basic properties of (singular holomorphic) codimension one foliations on a complex manifold. Then we survey of some old and recent results obtained when the ambient manifold is projective.

Guillaume Maurin (Université Pierre et Marie Curie – Paris VI), TBA

Fabien Pazuki (Københavns Universitet) Bad reduction of curves with CM jacobians

An abelian variety defined over a number field and having complex multiplication (CM) has potentially good reduction everywhere. If a curve of positive genus which is defined over a number field has good reduction at a given finite place, then so does its jacobian variety. However, the converse statement is false already in the genus 2 case, as can be seen in Namikawa and Ueno's classification table of fibres in pencils of curves of genus 2. In this joint work

with Philipp Habegger, our main result states that this phenomenon prevails for certain families of curves.

We prove the following result: Let F be a real quadratic number field. Up to isomorphisms there are only finitely many curves C of genus 2 defined over the algebraic closure of the rationals with good reduction everywhere and such that the jacobian Jac(C) has CM by the maximal order of a quartic, cyclic, totally imaginary number field containing F. Hence, except for finitely many examples, such a curve will always have stable bad reduction at some prime whereas its jacobian has good reduction everywhere. A remark is that one can exhibit infinite families of genus 2 curves with CM jacobian such that the endomorphism ring is the ring of algebraic integers in a cyclic extension of the rationals of degree 4 that contains F for some specific F.

Emmanuel Peyre (Université Joseph Fourier), The upgraded version of Batyrev-Manin program

From the point of view of Batyrev-Manin program about rational points of bounded height, accumulating subspaces hinder the equidistribution of points. Slopes à la Bost enable one to select points the distribution of which ought to be more uniform. This leads to a new version of empiric formulae which is compatible with the example of Batyrev and Tschinkel.

Hugues Randriam (Télécom ParisTech), Hermitian lattices and the analogy between number fields and function fields

In this introductory mini-course we will present a very basic notion from Arakelov theory, that of Hermitian lattices over the ring of integers of a number field, and stress the analogy we can draw between them and the theory of vector bundles over a complete curve.

In Lecture 1 we will focus on the rank 1 case (invertible modules, or line bundles). As an illustration, we will explain how classical theorems from algebraic number theory (finiteness of the class group, Dirichlet's units theorem) can be derived in this language.
(Main reference for Lecture 1: L. Szpiro. Degrés, intersections, hauteurs. In Astérisque Vol. 127.)

In Lecture 2 we will consider the higher rank case. We will introduce slopes and the canonical filtration, and discuss some of their properties.
(Main reference for Lecture 2: J.-B. Bost. Théorie géométrique des invariants et géométrie diophantienne. Handwritten notes from lectures in Orsay, 2008.)

Last, in Lecture 3 we will present a new extension of these notions, in the context of linear codes. In doing so we will try to highlight the links with number theory and algebraic geometry.
(Main reference for Lecture 3: H. Randriambololona. Harder-Narasimhan theory for linear codes. In preparation.)

Shun Tang (Capital Normal University), Localization sequence in arithmetic K-theory and the Lefschetz-Riemann-Roch theorem

In the context of Arakelov geometry, the higher arithmetic K-groups of an arithmetic scheme can be defined

via the Burgos-Wang's simplicial description of the regulator map, as suggested by Deligne and Soule. In this talk, we present a localization sequence of higher arithmetic K-groups for regular arithmetic schemes which are endowed with an action of the diagonalisable group scheme associated to a finite cyclic group. We shall also explain how to deduce from the localization sequence an arithmetic Lefschetz-Riemann-Roch theorem.

Dasheng Wei (Chinese Academy of Mathematics and Systems Science), Strong approximation for a normic variety

For an algebraic variety, strong approximation may imply the existence of integral points of its integral model. In this talk, I will introduce some varieties which satisfy strong approximation. It's a joint work with Ulrich Derenthal.

Chenyang Xu (Beijing International Center for Mathematical Research) Proof of Veys' conjecture

(joint with Johannes Nicaise) We discuss the proof of Veys' conjecture on the maximal order pole of the motivic zeta function. The proof relies heavily on minimal model program.

Fei Xu (Capital Normal University) Strong approximation with Brauer-Manin obstruction for varieties containing a connected linear algebraic group with compatible action.

There are several difficulties to extend strong approximation with Brauer-Manin obstruction for toric varieties to varieties containing a connected linear algebraic group with compatible action. For example, one has no the same type affine covering even over an algebraic closed field. Instead of explicit construction for toric varieties, we apply the descent theory developed by Colliot-Thelene and Sansuc and the rigidity of torsors under the groups of multiplicative type by Colliot-Thelene to prove such result. This is a joint work with Yang Cao.

Xinyi Yuan (University of California at Berkeley), On the Averaged Colmez Conjecture

The Colmez conjecture expresses the Faltings height of a CM abelian variety in terms of the logarithmic derivatives of certain Artin L-functions. In this talk, I will focus on an averaged version of the conjecture, which is proved in my recent joint work with Shou-Wu Zhang. Combining with the recent work of Jacob Tsimerman, the Andre-Oort conjecture for Siegel modular varieties is proved to be true.

Tong Zhang (University of Alberta), Linear series on certain arithmetic 3-folds

In this talk, I will state an effective upper bound of the global sections of arithmetic line bundles on arithmetic 3-folds fibered over arithmetic surfaces. It generalizes the previous joint work of Yuan and the speaker. It is still in progress.